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For Isabel in gratitude

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Preface

In its essence, this book is a suggestion to introduce certain systematic distinctions into modal philosophical discourse, nothing more. These distinctions are drawn from an analytical observation of our everyday use of language. They may be helpful for understanding this use of language. Once these distinctions have been adequately defined, they may also help in developing philosophical theory, in that their colloquial use may be translated into philosophical use, and thus may further diversify our options for theory design. Specifically, I hope that modal ontology may profit from the observations I make, but this remains to be seen.

In fact, I came upon the idea of modal pragmatics while working on a much more comprehensive and ambitious project about an ontology of happenstance. However, the proposal of modal pragmatics is completely independent of this further project, and I will not go into the latter in this book.

Being nothing more than a proposal, this text is directed not at a broader audience, but at researchers and advanced students in the field of modal theory. Its aim is not to instruct, but to contribute to a debate – or more precisely, to open a new field of discussion within the ongoing broader philosophical debate on modality. Because of this characteristic of my text, I have tried to keep it as succinct as possible and have made very few references to other authors. This book is not a grand, comprehensive *opus*, but rather an essayistic suggestion to do something new.

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Modal Pragmatics

When assessing a philosophical theory, we can distinguish between two fundamental points of view: the intrinsic and the extrinsic. These correspond, very roughly speaking, to two different aspects of truth as conventionally understood, namely to coherence and to correspondence. A theory can be judged by whether it is internally coherent, complete, economical, elegant, beautiful, fascinating, etc. But it can also be judged by "how much" and "how well" it explains. The quotation marks here are intended to indicate that in philosophical contexts it is usually unclear what the standard of this measurement is, and how its unit of measurement should be defined. This is due, among other things, to the fact that in order to accurately and systematically assess a theory, we need a further theory. The extrinsic assessment of a theory therefore always retains a certain degree of vagueness and subjectivity.

It is therefore understandable if many philosophical theorists stick primarily to the intrinsic criteria of theories. An extreme example of this is Hegel, who believed that the intrinsically perfect theory – that is, the completely successful execution of his own scientific philosophical project (to which he himself of course made no claim) – must automatically also be extrinsically perfect. But even today we see more and more theorists who place coherence, economy, elegance, etc. at the forefront of their discussions and treat the extrinsic qualities of their theories as secondary.

This seems to be particularly the case in modal theory. The object of explanation for most modal theorists today is logical necessity and possibility, or at most metaphysical necessity and possibility: necessity and possibility in the strictest sense. Without a doubt, this is a worthy subject of study. But if we look at our everyday dealings with modality and our ordinary talk of it, we notice that its object is very rarely logical or metaphysical necessity, nor is it just nomological or otherwise specified necessity. Typically, we talk about (specific) *conditional* necessity, and in most cases about necessity that is conditioned *within temporal nexuses*. However, it is precisely this kind of necessity and possibility that has only very marginal significance for most modal theorists.

Of course, the assessment of the extrinsic qualities of an explanatory theory always depends on what we want to be explained and how we want it to be explained. But if we assume that it is desirable for a modal theory to conceptually explain the use of modal language that we actually see in everyday life, then logical-metaphysical modal theories only *explain* to a small extent. It therefore seems appropriate to first consider how necessity, possibility, impossibility and nonnecessity are discussed in ordinary linguistic practice.

Necessity and Possibility in Ordinary Language

In everyday life, we speak in various ways about necessity and chance, about possibility and actuality. This linguistic practice can be examined in various ways. One can examine what logic underlies it and strive to formalize it. One can try to explain its semantics and develop models for these, such as possible world semantics (PWS). One can consider what ontology could underlie it – in what sense, e.g., the counterfactually possible is real or not. One can examine what different areas of application modal expressions have: one can, for example, distinguish logical modality from metaphysical, nomological, doxastic, deontic, juridical and other modalities.

In the following I will try to clarify another aspect of what we mean when we use modal expressions. It seems to me that, in addition to the differences mentioned, there is also a differentiation in our modal linguistic practice that we could call "operational" or – on the basis of the common classification of the various aspects of language into semantics, syntactics and pragmatics – "pragmatic", and which stands in relation to the practice of logical reasoning. This differentiation seems to me more fundamental than all the others because it affects our understanding of modal expressions as such, that is, it has to do with how modality "works" in the first place. Since I want to focus on this aspect, I will leave aside the other aspects of modality in this book. In particular, I will only address questions of modal logic in passing.

I *will* refer to PWS in my discussion. However, this will only serve as a means of elucidation; I do not associate any ontological or other thesis with it (on the contrary, I personally think that it is inadequate to

articulate a modal *ontology* on the basis of PWS, but I will not elaborate on this point here). It will therefore not be necessary to go into the different theories of possible worlds that have been developed over recent decades. It will suffice for the following discussion to conceive of a *possible world* as that with respect to which every ("normal"[1]) descriptive proposition is either true or false and, furthermore, such that all propositions that are true with respect to it are logically consistent with one another.[2] The *totality* of possible worlds therefore includes that to which *all* consistent constellations of truth or falsity, *of all* descriptive propositions, have their respective reference. We can imagine possible worlds as maximally consistent states of affairs, or as maximally consistent individuals (that depends on the ontology we choose), or as maximally consistent ideas about these. What is crucial is that, on the basis of our conception of possible worlds, the necessity of a statement can be understood as its truth in all possible worlds, and the possibility of a statement can be understood as its truth in at least one possible world.

If we develop the operational or pragmatic aspect of modality, then we can subsequently also develop models of modal semantics and syntactics, and then also develop models of modal ontology that take up the different theoretical options that modal pragmatics provides. These models will not be discussed further in this book, however.

In order to simplify my formulations, as well as for aesthetic reasons, I will often speak only of necessity in the following when I mean modality in general.

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1 To simplify things, we could add here: any "normal" *nonmodal* declarative sentence. Of course, we can also talk about the world-reference of modal propositions, that is, if we want to discuss the access relations of possible worlds to one another. But we need not discuss this issue further in this context, because we only want to introduce a preliminary, popular, and naive concept of a "possible world".

2 For example, with respect to every possible world it is either true or false that Peter exists; and if he exists in a world, then it is false with respect to that world that Peter is not spatially located, because that is incompatible with the existence of Peter, see below. (On the other hand, the latter sentence is true if Peter does not exist, at least according to the logical interpretation commonly used today – see below – because it then says that it is not the case that Peter exists while being spatially located.).

Division

To make it easier to understand what follows, I will indicate up front the systematic structure of the pragmatic modal theory that I want to develop: Pragmatically, *formal* necessity (§1) can be distinguished from *material* necessity (§2). The former can be further divided into *inferential* (§1.1) and *axiomatic* necessity (§1.2), and the latter into *unconditional* (§2.1) and *conditioned* necessity (§2.2), where conditioned necessity can be further divided into *apodictically conditioned* (§2.2.1), *problematically conditioned* (§2.2.2) and *factually conditioned* necessity (§2.2.3), on the basis of Kant's distinction between the three moments of modality as a function of judgment.[3] The other forms of necessity could also be further subdivided, but that is of no further interest in our context.

The pragmatic differentiations correspond in a certain way to some that are common in traditional modal theory. But these correspondences are superficial and imperfect. The differentiation made here cuts across the traditional ones, as will become clear from the following explanations.

My immodest claim is that, with modal pragmatics – as far as I can see – I am opening up a new field of modal theory. That is, no coherent theory has yet been presented in this field adequately, consistently and systematically. The following presentation may therefore contain various errors, as is almost always the case with theories that establish a new field of research.

In some places I will use formal logical formulations. This only serves to provide greater clarity; I associate no argumentative claim with its use. These formulations are only intended to help to clarify ordinary language usage and to make it easier to understand; I do not aim to replace ordinary language with a better language. Therefore, in some cases, these formal formulations will seem unorthodox, in that they simply follow our normal way of speaking.

Generally speaking, the distinctions I want to make may seem "unorthodox" from the point of view of formal modal logic, in that some of them are difficult or even impossible to articulate in some formal languages. However, they can all be articulated on the basis of PWS, within the

3 Cf. Kritik der reinen Vernunft, B 95.

grammar of ordinary language. That is, PWS provides all that we materially need to account for the modal pragmatic distinctions which I want to explain.

To illustrate my view of formal logic in general, I would like to quote a rather lengthy section of Craig Delancey's *Concise Introduction to Logic* (2017), where he introduces proper names:

> Recall that we want our language to have no vagueness, and no ambiguity. A name would be vague if it might or might not pick out an object. So, we will require that each name pick out an object. [...]
>
> We might decide also that each thing that our language talks about has only one name. Some philosophers have thought that such a rule would be very helpful. However, it turns out it is often very hard to know if two apparent things are the same thing, and so in a natural language we often have several names for the same thing. A favorite example of philosophers, taken from the philosopher and mathematician Gottlob Frege (1848–1925), is "Hesperus" and "Phosphorus." These are both names for Venus, although some who used these names did not know that. Thus, for a while, some people did not know that Hesperus was Phosphorus. And, of course, we would not have been able to use just one name for both, if we did not know that these names pointed at the same one thing. Thus, if we want to model scientific problems, or other real world problems, using our logic, then a rule that each thing have one and only one name would demand too much: it would require us to solve all our mysteries before we got started.[4]

This very basic introductory explanation exemplifies a very basic problem that formal languages have: If they require us to solve all our mysteries before we get started, then they may not be very helpful for what we intend to do. Formal languages trade clarity and distinctness for complexity and concreteness. However, in doing this, they may eventually get too "poor" or too "strict", or too restricted to formulate the problems we are interested in. We can also put it the other way around: It is not helpful if a formal language removes all mysteries by forbidding us (or more precisely, by not making it possible for us) to formulate them. However, it is not, in itself, a great achievement to build a language where there is "no vagueness, and no ambiguity". This is fairly simple. The question is whether such a language may be helpful in solving or at least in discussing the "mysteries" we encounter.

4 Delancey 2017, 148–149.

I am convinced that the distinctions I want to draw can be articulated without (abnormal[5]) vagueness and ambiguity in ordinary language, by relying on PWS, and this is what I will try to do in the following. It certainly would be desirable for these distinctions to also be articulable in a formal language, because that would probably be helpful for further discussions on the basis of these distinctions. However, I will not be able to fulfill this desideratum in this book, and the validity of what I will say does not depend on its fulfillment.

5 The *normal* vagueness and ambiguity of all expressions stems from the fact that no expression can be perfectly precise and distinct. A simple initial argument for this is that precision and distinction require relationality, which leads to an infinite regress.

1. Formal Necessity

1.1 Inferential Necessity

We often use modal language to express the necessity in logical inferring, for example: "All of Anna's sons are married. Peter[6] is a son of Anna. Therefore, necessarily, Peter is married." Anyone who speaks like this obviously does not want to say that being married is an essential property of Peter, nor do they want to say that it was a natural necessity for him to get married. They also do not want to assume (regarding the first premise) that all of Anna's sons are *necessarily* married. It is therefore completely inconceivable for anyone who speaks like this to mean that there is any necessity in the matter, i.e., a *material necessity*. They simply want to express the coerciveness of their logical inference. This necessity is often called "logical" or "*necessitas consequentiae*". I would like to call it "inferential", because this better expresses the place it occupies within the framework of the system developed here. It will become clear why.

It seems that this is the necessity we mean when we speak of necessity in everyday epistemic contexts – let us call this "epistemic necessity" (which is different from "doxastic modality", see below). When Sherlock Holmes says that *necessarily* the gardener was the murderer, he does not mean to say that the gardener was forced to murder by the laws of nature, or even by logical-metaphysical laws, that is, that he commits the murder in question in all possible worlds. What Holmes means is that, on the basis of the premises known to him, and with the rational capacities he has, he necessarily *comes to the conclusion* that the gardener was the murderer.

Epistemic necessity in this sense – i.e., as the expressions "necessarily" and "possibly" (or "must" and "can") are used in epistemic contexts – is to be distinguished from so-called "doxastic modality" in discussions of modal logic.[7] Doxastic modality does *not* speak of necessity and possibility in the narrower sense (although these words are often used to express

6 The use of "Peter" in my examples is, among other things, a reference to Jean-Paul Sartre.

7 Which is in turn distinct from talk of epistemic modality in linguistic discourse.

it, as are the words "must" and "can"), but instead understands the necessity operator (loosely speaking) as "one knows that …" (or "it seems that …") and the possibility operator as "one does not know (for sure) that not …". Under this interpretation of modality, one can, of course, model the logic of exactly this way of speaking (that is, of "one knows that …" and "one does not know that not …"). But this does not model our modal talk in examples such as "necessarily the gardener was the murderer", because we cannot always replace one way of speaking with the other. For example, we know that Heidelberg is on the Neckar; but we do not say in this context: "*Necessarily* Heidelberg is located on the Neckar." So, when we say that necessarily the gardener was the murderer, that does not mean simply that we know that the gardener was the murderer, but rather that we have come to this knowledge by valid inference.

Doxastic modality can be modeled in PWS as follows: We form the set of all possible worlds that are consistent with what "one" knows (whoever "one" may be), or what a particular person knows. Then what is the case in all these worlds is doxastically necessary (with respect to "us" or the person in question) and what is the case in at least one of them is doxastically possible. Consequently, this results in the speech practice just criticized: If one knows that Heidelberg is on the Neckar, then *necessarily* Heidelberg is on the Neckar, because this is the case in all the possible worlds that have been selected.

However, this misunderstanding clarifies a point that will be important later: If we select a set of possible worlds under the criterion of the existence of a certain state of affairs (or a state of affairs that includes this state of affairs as a partial state of affairs) within them, then this state of affairs is trivially "necessary" in the sense of the quantification over this selection. This means that such "necessity" can have no meaning at all, because according to this schema, any (possible) state of affairs is necessary. The same applies, of course, if we make a selection of possible worlds under the criterion of the truth of the premises of a certain inference in it – for example, if we select all worlds in which it is true that all of Anna's sons are married, *and* in which Peter is a son of Anna, then trivially, the conclusion also is "necessarily" true regarding these worlds, namely that Peter is married. But this "necessity" expresses nothing at all, neither logical necessity nor inferential, epistemic, or any other necessity. It is simply a consequence of the fact that with regard to the selected

worlds – trivially – the premises are necessarily true, and that if all the premises are true, then necessarily the conclusion is not only true, but necessarily true (that is, according to the so-called axiom schema K: If, necessarily: A implies B, then necessarily-A implies necessarily-B).

This is important because in what follows I want to present a type of necessity that, formulated in PWS, in fact has to do with making a selection of possible worlds according to the criterion of the existence of a certain state of affairs in them. Such an approach only makes sense if the necessity that we are talking about is not already given by this selection alone, but instead only arises from it together with something else, typically a *nomologically* necessary premise.

Colloquial epistemic necessity (which is different from doxastic necessity) can be called by this name – in contrast to other forms of inferential necessity, if one wants to name it specifically at all – because it is bracketed by the clause "as far as we know …". But if we look more closely, this clause does not actually bracket the modality in question, but rather the premises (as nonmodal, i.e., as purely actual affirmations) or the inferential transition from these to the conclusion.[8] This bracketing, consequently, is also inherited by the conclusion. However, the fact that we speak of necessity and possibility in the context of what we know has nothing to do with "as far as we know …" as such, but rather with the fact that *within* such epistemic contexts we draw logical conclusions.[9] For example, if a witness directly observed the gardener committing the murder, then they will not say: "Necessarily the gardener is the murderer",

8 The subjective limitation of our knowledge may not only lie in the content we know (because we are not omniscient), but also in our inferential capabilities: we are not (yet) able to understand all the implications of what we (already) know. This latter is very evident in mathematics: we have not yet found all proofs that there are that are based on the axioms we already know (or have established, depending on one's metamathematical beliefs).

9 On the contrary, we colloquially cancel the formula "as far as we know …" precisely when we have reached a clear conclusion. We say: "As far as we know, either the maid or the lawyer or the gardener could have been the murderer." Obviously this only applies "as far as we know", because *in fact* it was exactly one of the three, and the other two cannot have been the murderer. However, once we have come to the conclusion, through a process of eliminating possibilities, that the gardener must have been the murderer, we typically no longer say: "as far as we know …", but simply affirm: "The gardener must have been the murderer."

but will simply say: "The gardener is the murderer."[10] They have no reason to speak modally because they have not arrived at their knowledge through an inference. Of course, the formula "as far as we know …" has to do with the use of modal language in that, in cases like the one presented, we rely on inferential procedures precisely because our direct knowledge of the facts is limited. But the modal character of the relevant statements still stems from the process or the pragmatics of inference. If our knowledge of a fact is limited, but we have not arrived at it through inference, then we use the formula "as far as I know …" without modal expressions: "As far as I know, Peter is married to Mary." So I heard from a friend a long time ago. This information may have been unreliable; Peter and Mary may have since divorced, etc.

If "it is necessary that …" in its inferential sense means: "it unequivocally follows from a valid inference that …", then, in their inferential senses, "it is not necessary/it may not be the case that …" mean: "it does not clearly follow from a valid inference that …", and "it is possible that …" means: "it does not follow from a valid inference that not …". In the case of epistemic possibility, this fact is still bracketed by "as far as we know …". "The Goldbach conjecture is possibly true" in its epistemic sense means: "As far as we know, it is not the case that it unequivocally follows from a valid inference that the Goldbach conjecture is not true",[11] or more simply: "We do not know of any proof that the Goldbach conjecture is false." That is, we in fact talk about the *necessity of the inference*, which *as far as we know* is not given, precisely in the sense of inferential possibility.

It seems like epistemic possibility is also what we have in mind when we talk about *theoretical* possibility, as in the following example: If an agnostic admits the possibility of God's existence, then (if they are consistent) they do not admit the *material* possibility of God's existence, i.e., God's existence in some possible world. Rather, they only admit its

10 In a sense, in this context the expression "The gardener was the murderer" is stronger than the expression "Necessarily the gardener was the murderer". The eyewitness's testimony carries more weight than the detective's conclusion.

11 In this case, the constraint "as far as we know …" does not bracket the premises, but rather the *whole* of the premises and the possible conclusion: we do not know of any inferential link from the premises (which we may very well know) to the falsity of the Goldbach conjecture. However, this bracketing does not affect the mechanics of inferential necessity.

theoretical possibility. Because if God exists, this must be a necessarily existent being. It cannot be the case that God exists only in *some* possible worlds (eventually including our own) and not in others. (This could only be the case, if at all, with regard to nonuniversal, nonabsolute gods, maybe in the context of some form of polytheism.) However, as usually understood (i.e., leaving aside questions of modal access and other problems, which, intuitively, should have no bearing on this question), what is *possibly* necessary, is *in fact* necessary, because, in PWS, "x possibly exists necessarily" means: "In some world there exists x, which exists in all worlds." Therefore, when we talk in ordinary language of the possibility of God's existence, we typically talk about a form of possibility which has nothing to do with possible worlds. And this is exactly what characterizes inferential possibility. If we say: "Possibly God exists", then this means that there is no proof of God's nonexistence. Notice that this sentence can be stated in an epistemic context, meaning: "We do not know of any proof of God's nonexistence." However, it may also be the case that there is, *in fact*, no proof of God's existence, and no proof of God's nonexistence either; at least that is what some philosophers like Immanuel Kant have argued. Of course, this is a strong thesis to defend (perhaps even stronger than the thesis of God's existence, or of God's nonexistence): "There is, *in fact*, no proof of either the existence or the nonexistence of God." But, as far as we know, this thesis is (again) theoretically possible: There is (*pace* Kant) no proof of the provability or unprovability of God's existence or nonexistence, at least none that we know to be valid with sufficient certainty.

Epistemic necessity and possibility look subjective, because they have to do with our knowledge or lack thereof. This is correct insofar as this necessity is not an objective one. When Holmes says: "Necessarily the gardener was the murderer", it is not *factually* necessary that the gardener committed the murder. (Traditionally this difference is formulated as that between *necessitas consequentiae*, which I have already briefly mentioned, and *necessitas consequentis*. More on this below.) This nonobjectivity or "irreality" of epistemic necessity becomes even clearer in the case of statements of possibility that later turn out to be false: "As far as we know, the lawyer could also have committed the murder", says Holmes. He then finds out that the lawyer was already on a ship bound for South Africa at the time in question. "The lawyer could not have been

the murderer", he concludes. Of course, only the epistemic modal situation has changed, not the factual situation.

But if it is true that epistemic necessity essentially represents inferential necessity, then this *necessity* is itself not subject-dependent at all. Only the epistemic bracketing of the premises or of the inferential nexus (i.e., one's knowledge of that nexus) is subjective, and this is then inherited by the conclusion or, in the case of epistemic possibility, one's lack of knowledge of a conclusion to its negation. But inferential necessity does not concern premises and a conclusion as such, but rather the passage from the former to the latter. However, the necessity in the steps of the inference is entirely objective. It has nothing to do with any subjective practice as such (and of course not with any empirical mental conditions or mental processes – inferential necessity is not some kind of "mental necessity"), even if we typically know it from our subjective practice of reasoning on the basis of premises that are known to us. Inferential necessity concerns the "transition as such" from the premises to the conclusion. As such, and as will be shown below, inferential necessity is the fundamental form of necessity – at least, viewed from the perspective of abstraction, or from the point of view of formality. All other types of necessity must presuppose it, from the perspective of abstraction.

1.2 Axiomatic necessity

We often use the word "necessary" to express the way in which the ultimate, general principles that we presuppose in our reasoning are valid. We may call this necessity "axiomatic" (if we understand the term "axiom" broadly). For example, we say that everything is necessarily identical with itself. And we say that bodies with mass necessarily attract each other. Now, there are various contexts in which we use modal expressions: logic, metaphysics, physics, morality, the legislation of a particular country, the rules of a particular game, and so on. With regard to all of these nexuses we speak, *de re*, of specific axiomatic necessity.

In the context of PWS, the *original* meaning of axiomatic necessity is not intramundane necessity (i.e., material necessity), but world-determining or "world-selecting" necessity. It tells us what kind of world we are in – or what kind of country we are in, what kind of game we

are playing, etc. If we understand statements that are intended to indicate axiomatic necessity in the sense of intramundane necessity, then they become tautologically empty – just as empty as the statement: "Heidelberg is necessarily located on the Neckar", where this statement is based on the selection of those possible worlds in which Heidelberg is located on the Neckar. Correspondingly, when we say: "All massive bodies that are together in space necessarily attract one another", we obviously do not mean to say that this is the case in all possible worlds, i.e., in all logically possible worlds, because the laws of nature (as far as we know) are not logically necessary. So in order to express what we want to say in PWS, we must specify: "In all nomologically possible worlds …", that is: "In all worlds in which the same laws of nature are valid as in our world, all massive bodies that are together in space attract one another." However, nomologically possible worlds are defined, among other things, by the fact that the law of gravity is valid in them. It is therefore unsurprising that no states of affairs which contradict this law can be found in such a world.[12] The statement that the law of gravity is valid in all nomologically possible worlds is trivially analytic: it explicates the term "nomologically possible world".[13]

This perhaps becomes even clearer when we consider axiomatic necessity in the context of games. We can say, e.g., that in a game of chess, the rooks necessarily move exclusively orthogonally. Now, the players can change this rule and agree that in their next game the towers can also

12 I do not want to go into the question of the possibility of miracles here. My short answer would be that if miracles are possible in a world, then this is part of its nomological setup. In this case not only the laws of nature are valid, but also, e.g., the law that God can act against the laws of nature.

13 By "in the trivial sense", I want to say that the tautology is "direct", as in the case of "All bachelors are unmarried" or "All triangles have three angles": the predicate concept is directly contained in the definition of the subject concept. This specification is necessary because many believe that the propositions of mathematics and logic are analytic throughout. But, of course, many of those propositions are analytic in a nontrivial sense, because it is not directly contained in the definition of a triangle, e.g., that the sum of its angles (in Euclidean space) is always 180°. Of course, it may be disputed what a definition directly contains, and what it indirectly contains or implies – and whether triangles cannot also be defined directly by the sum of their angles. But it seems to me that this distinction is sufficiently clear for our current argumentative context.

move diagonally. But then, we will say, they are no longer playing chess, because the game of chess is constituted by its rules of play.

Sentences that speak of necessity in this sense are nontrivial in *definitions*: In a chess manual, the rule as to how rooks are allowed to move is nontrivial. Sentences like "The law of gravity is valid in all nomologically possible worlds" are not definitions, but they are not trivial in the corresponding typical linguistic contexts because – unlike in games – we do not know the laws of nature from the outset, nor can we find them in game instructions. Rather, we must find out about them. We do not yet know all that the concept of a "nomologically possible world" encompasses.[14] Physics has discovered a lot about this but is far from having concluded its research on the laws of nature. Such sentences are nontrivial precisely when they are understood as explicating the concept of a "nomologically possible world", that is, as a (partial) explication of what such a world is. They are *then* tautologically trivial (but by no means superfluous, as we will see) if they are understood as statements about intramundanely necessary states of affairs, because in order to explicate this necessity we must use a concept that already contains the determination of these states of necessity: the concept of (consistent) nomological determination.

The Systematic Indispensability of Formal Necessity

Due to the triviality of axiomatic necessity on the one hand, and the apparent subjectivity of inferential necessity on the other hand, as well as the formality of both, it may seem sensible at first glance not to

14 We can therefore call the concept that we (currently) have of nomologically possible worlds, or of the (total) laws of nature, a "preliminary concept". We typically form a preliminary concept through indexical reference: "the concept of the totality of those natural laws that apply in *this world of ours*"; "the natural kind of animal that is crawling around on the ground *in front of me*". Of course, we do not *know* these concepts yet (we are not yet "acquainted" with them), but they are clearly "fixed" through the respective indexical reference. Evidently, many of the concepts that we use every day are preliminary concepts in our (current) usage of them, i.e., we do not yet know them completely and do not know what (exactly) they do and do not include or imply.

consider these two as forms of necessity in the strict sense, because they do not represent necessity at all with regard to "the matter" or "things in themselves" – i.e., they represent no material necessity. In the case of what I called "axiomatic necessity", talking about necessity can even lead to misunderstandings, because we can certainly discuss the objective, material necessity of natural laws, be it of individual laws or the laws as a whole – e.g., whether it is metaphysically or even logically necessary that the law of gravity is valid. (In this case the *material necessity of the law of gravity itself* is discussed, with regard to *all* possible worlds.) Therefore it may seem appropriate, at first glance, to reduce talk of necessity to talk of validity (or truth) in the two cases described. In our previous example it is *true* (as far as we know) that the gardener was the murderer; and the laws of nature are *valid* in our world (which in PWS is equivalent to: our world is one of the nomologically possible worlds).

As long as this reduction is supposed to be a purely terminological decision, there is of course nothing wrong with it. But the *validity* of logic, and of the laws of nature, is directly connected with the *necessity* that we unquestionably understand as such, that is, with intramundane, material necessity. The validity of logic and natural laws manifests as a necessity *within* our world. The so-called Necessitation Rule expresses this connection when understood broadly. Narrowly understood, this rule states that if a statement *p* is logically derivable, then *necessarily p* is also logically derivable. Broadly understood, it states this connection in relation not only to logical derivability, but also to derivability from axioms in the sense presented (whereby the rule trivially also necessitates the axioms themselves).

However, this rule is required because special laws such as the laws of nature often function as premises for conclusions about the necessity of individual states of affairs within the world, that is, according to the axiom schema K: "$\Box(A \rightarrow B) \rightarrow (\Box A \rightarrow \Box B)$", which underlies all so-called normal systems of modal logic. But if these premises were not necessary, then we could not infer necessity from them. However, we do this all the time, every day. We say, for example, that when an atom splits, it will necessarily emit a certain amount of radiation. We do not just want to say that it will *actually* (in the actual world) do this, because, purely theoretically, it could actually emit radiation by chance. Nor do we just want to say that it will *actually always* do this, because that too could be

a matter of coincidence. (Perhaps it is always the case in our world, but not in all nomologically possible worlds.) What we want to say is that an atom *necessarily* emits radiation when it splits. And we could not do so (according to K) unless all the premises that we implicitly draw on in our conclusion were (materially) necessary, that is, the premises stating the laws of nuclear force and those stating the relevant essential properties of the atom.

We must therefore insist that the principles that determine the world, such as the laws of nature, have material necessity *within the world* (we must therefore adhere to the Necessitation Rule) – as is quite inevitable when the corresponding world is formally determined by this principle. But we cannot *reduce* the validity of such axioms to material necessity, because then it loses its meaning. (It would then only be "necessary" in the sense in which Heidelberg is "necessarily" on the Neckar in our above example.) The fact that axiomatic necessity and therefore *a fortiori* also inferential necessity is presupposed by material necessity, and cannot be reduced to it, can be seen as follows: The set of possible worlds in which a certain natural law is valid is not identical with the set of possible worlds in which no state of affairs occurs that contradicts this law of nature, because there are possible worlds in which a certain law of nature is not valid, but in which, as it happens, no states of affairs occur that violate this law. Otherwise, talk of particular laws such as the laws of nature would be completely trivial, because it could then only mean that in all worlds in which no states of affairs occur that do not conform to the law in question, no states of affairs occur that do not conform to that law. (The necessity of natural laws would thus reduce completely to "Heidelberg necessity".) According to this latter schema, we can form the set B of all worlds in which there is no cow that is not called Berta, and then say that in all B-possible worlds it is necessary that all cows are named Berta. The selection of those possible worlds that constitute the set of nomologically possible worlds (or other types of possible worlds) cannot be determined by the (nonlawlike) contents of this world. It must instead be determined by the *validity* or *formal necessity* of the relevant laws. (The selection cannot be determined extensionally. It must be determined intensionally.) This validity is formulated via the use of modal speech according to §1.2 – but not by formulating the necessity of the laws of nature as intramundane necessity.

If a set of worlds is constituted on the basis of axiomatic necessity, then – as I have said – we can of course also talk about the intramundane occurrence of the correspondent lawlike state of affairs regarding these worlds – for example about the fact that in all nomologically possible worlds, all massive bodies attract each other. Although this is trivial in itself, it can make sense in the context of an inference. Nevertheless, there remains a fundamental difference as to whether we talk about the necessary *existence of this state of affairs* within the world (that is, about its occurrence in all nomologically possible worlds), or instead about the *validity of the law of gravity* for all nomologically possible worlds – that is, whether we are talking about material or formal necessity. On the other hand, however, it seems sensible to speak of *necessity* in ordinary language in both cases, because logical and nomological validity are directly connected to intramundane necessity via the Necessitation Rule. Loosely speaking, the talk of formal necessity is justified if we read the Necessitation Rule backwards, that is, in such a way that inferential or axiomatic validity is characterized by the fact that it is necessary within the world. However, it seems that neither inferential nor axiomatic validity can be characterized (in this respect) in any other way than by resorting to the Necessitation Rule. More precisely, inferential or axiomatic validity is characterized by *outright* universality (whereby outright universality can be restricted, e.g., to all nomologically possible worlds). In theory, we can consider actuality without modality, e.g., we can discuss nonmodal logic. Then the outright universality of this validity is of course not represented modally, but only as actual. But as soon as we expand our nonmodal logic into a modal logic, this validity *qua* outright universal automatically also extends to this expansion; so to speak, to the space of determination that is opened up by this expansion. And this is exactly what the Necessitation Rule expresses. Logical space is the space of the possible (in PWS: of all possible worlds). The validity of logic in the entire logical space (which the Necessitation Rule articulates) is therefore trivial, but it still characterizes logic decisively: as that which has trivial or *eo ipso* necessary validity. (An analogous point applies to natural laws and other axioms when restricted to the appropriate selection of possible worlds.) And that is why it makes sense to articulate the (specific type of) validity of logic – including the validity of specific axiomatic systems – as formal *necessity*.

We can also say that this usage of language expresses, so to speak, the "necessity" of the Necessitation Rule itself: "It must be the case that this rule applies" – formal necessity must precipitate materially. Consequently, if formal *validity* precipitates materially as *necessity*, then it must also be articulated *formally* as necessity or necessary validity. Because if (in any theory) the space of the modal is opened up, then the connection between validity and the scope of validity is no longer directly given, because there are now different possible scopes (in PWS: different possible worlds) in which something may or may not apply. If there are *different* possible scopes of validity, then this *eo ipso* opens up the (theoretical) possibility of limited validity: of validity not in relation to all possible scopes of validity (in PWS: all possible worlds), but only to some. Therefore it is of particular significance when some specific validity, in fact, applies to all possible scopes of validity, and this particular significance is articulated by formal necessity. Therefore it makes sense to say that logic is *necessarily valid*, and not just that it is valid (or just *universally* valid).

But it also makes sense to speak of necessity with regard to validity that does not simply exist with regard to *all* possible scopes of validity (i.e., logical and mathematical validity), but is rather a case of merely axiomatic validity (such as that of the laws of nature), because we also speak of validity in relation to limited scopes within the world we talk about – for example, about the validity of a country's laws or of the rules of a game. The *necessity* in, e.g., "nomological-axiomatic necessity" states that the validity of the axioms in question refers to our world as a whole, namely it constitutes this world: the laws of nature determine, among other things, what kind of world our world is (as such). This means that we cannot exit the scope of this validity, unlike the scope of the laws of a specific country or the rules of a specific game, because we would have to exit our world to do so.

This relationship can then be further generalized or relativized: It *always* makes sense to speak of axiomatic necessity when the area of validity that it constitutes (possibly) gives room for limited, particular validity (that is, within this area) – and not only when this former area represents a world, or our world. For example, a certain field of music is constituted by the validity of the diatonic-chromatic-enharmonic tonal system. If a musician plays a note outside of this system within this field,

then he is playing a wrong note. But within this field there are many subfields (classical music, jazz, pop, etc.), each of which has additional, specific rules. To put it the other way around: When we speak of particular validity, we implicitly always presuppose general validity, that is, the validity that constitutes the space of our speech, that is, of (the givenness of) what we talk about. Because without such general validity (and hence without any grammar) we could not speak at all, and we could speak about nothing at all (for it is impossible to speak about something that is completely disordered). This *possibility-conditioning* validity fundamentally and always has a different status than the validity that may show up as a particular one within a specific field, or the validity that we may be talking *about* within this field. As I have said, this distinction is entirely a relative one: What is possibility-conditioning validity in a certain context may simply be particular validity in another, more comprehensive context. But this relativity does not abolish the fundamentality of the distinction as such, because whenever we speak of validity, we must already assume validity, which makes our speaking possible in the first place.[15]

That is, intramundane, material necessity always presupposes formal or transcendental[16] necessity because it can only articulate itself on this basis. But conversely, formal necessity derives its meaning *as* necessity solely from material necessity. Without this it does not represent necessity, but merely validity. But this validity remains empty as long as it is not validity *for* something. And in this "being valid *for*", validity presents itself as necessity. Therefore it is no contradiction if, although formal necessity is presupposed by material necessity as a condition of its possibility, the latter (or even a particular form of the latter) may represent the original type of necessity in the ontological sense (as I would like to argue myself, although not in this book). The formal necessity of logic and world-constitutive laws originally, purely on its own, does not represent necessity at all, but rather empty validity.

15 This is why we must ultimately base axiomatic formal necessity on inferential formal necessity: without any given (or pragmatically functional) determination and order we cannot even formulate axioms.

16 Here in the "generalized" sense, that is, not only in the sense of a "condition of the possibility of our cognition", but simply in the sense of a "condition of (any) possibility (at all)".

Since, according to the Necessitation Rule, the laws of logic and nature are *also* intramundane states of affairs, one could perhaps come up with the idea of considering them *solely* as intramundane states of affairs, for the sake of theoretical economy. Obviously there are laws that exist only intramundanely, such as game rules or legal systems. So why should the laws of nature, or even the laws of logic, not also be states of affairs that obtain only *in* the world, or in the worlds? For logic, the absurdity of such mundanization is quite obvious. It would turn out that logic is valid in our world just because it exists in our world. Of course, a given thing in *one* world cannot, as such, have any meaning for other worlds, except for being part of a *possible* world (which, from their perspective, is counterfactual). Such validity of logic in our world could not mean that *all* possible worlds are logically possible worlds, or that logical space is the space of all possible worlds. Of course, we could select the "logically possible worlds" according to the content criterion of the existence of the state of affairs of the validity of logic in them, and then quantify over them in order to speak of logical necessity. But then worlds could *also* be possible in which logic is not valid. Conversely, the fact that logic applies in our world would be completely coincidental. There would then be worlds in which this state of affairs exists and others in which it does not exist. Ours happens to be one of the former. However, the problem then arises that we cannot know for sure whether our world, in fact, is one of the former. We could only say: "As far as we have observed so far, the laws of logic are not violated in our world." As with the conformity to natural laws discussed above, it could now also be the case that our world only behaves in accordance with logic by chance, so that the validity of logic would not be a state of affairs in it at all. At the very least, this does not correspond to our usual intuitions about the validity of logic. Logic, as most people would intuitively say, is not just factually valid, it is necessarily valid. It is simply not possible for there to be a world in which logic is not valid.

However, this way of putting things implies a presupposition problem: If logic determines what is (logically) necessary, possible, impossible, and nonnecessary, then what determines the necessity of logic itself? To claim that logic itself is *logically* necessary is obviously circular. The answer is: logic represents the minimal requirement for determining necessity and possibility, or for the semantics of modal concepts. Without logic there is no differentiation of *possibility at all*: the negation of possibility

(or necessity, etc., depending on what you want to use as your basis for modal theory); i.e., not-possible, possibly-not and not-possible-not, could no longer be distinguished from one another, because then there would simply be no more restriction on the possible. But this would make the concept of possibility meaningless. Without modality, however, semantics itself is no longer possible, because semantics requires that there is not just bivalence (e.g., what is blue is not not blue), but also quadrivalence (e.g., what is blue cannot also be yellow, but can also be square, warm, etc.). Beyond logical space there is *nothing*, or nothing definite. There can only be *something* in logical space. This is why the validity of logic cannot *just* be an intramundane fact – although, as I have said, it *also* is.

For nonlogical axiomatic necessity, the requirement of formality is not so easy to see. At first glance there seems to be no argument against the idea that the laws of nature are merely states of affairs *in* the world. To get clear about this, let us look at the example of games again. Games are *constituted* by their rules of play. Anyone who disobeys these rules will no longer play the game in question. But some games apparently have no rules. Other games allow rule violations ("cheating"); others allow their rules to be changed or reinvented within the game. Hence, apparently, we could conceive of a game in which everything is allowed, including establishing rules *in* the game. In such a game, the rules would be immanent, but not constitutive. Let us think about what such a game would look like. The rules of a game, broadly understood, include all its restrictions, including regulations about when the game begins, when it ends, and who plays. Participation in a game does not have to be based on consent. Children may, for example, let their father take part in their game without asking him, and without his knowing: he is the evil monster that you must hit with the water balloon without being discovered. So what would a game be without any restrictions? It would be the global game: everyone and everything would participate in it in any way they wanted, from time immemorial and throughout the future (including establishing new rules, or even playing other subgames within the game). Individual players could try to impose limitations on the game by introducing new rules, for example regarding time or space, or regarding its participants. But because there are no rules for introducing new rules, any other player could refuse to accept these rules and introduce other rules themselves.

This means that as a whole, this game is no longer distinguishable from the world; it is no longer possible to say specifically what is valid in this game and what is not, because there are no rules for introducing rules or breaking rules. But this means that this game loses the meaning of being a game in any distinct sense. So at least *some* constitutive rules (in the broadest sense) are required for something to be a game in any distinct sense, even if these rules themselves may allow for *immanent* rules (including the creation of such rules within the game).

Now let us consider what a *world* would look like that had no constitutive limitations. It may well have intramundane restrictions, but there would be no restrictions on the introduction, removal, scope and limitation of the scope of such restrictions. So, for example, it would be allowed to introduce a universal restriction (a universal rule), but it would also be allowed to introduce a "universe" in which this restriction does not apply. In particular, there would be no restrictions on spatial and temporal dimensions in this world. There would also be no restrictions on introducing restrictions on certain dimensions. So, for example, there could be three specific spatial dimensions within this world, together with a temporal dimension, which would be internally subject to certain natural laws and contain certain objects, but which would not have any connection with anything in the other dimensions. But this means that this completely unrestricted world would contain unlimited worlds within it, that is, all possible worlds.[17] It would therefore be identical to the space of possible worlds. But with this, it would no longer be a world.

In order for a single world not to be all possible worlds, we must demand that everything in it stands within a definite (determinate) nexus (an indefinite or indeterminate nexus would be as good as none). This may (theoretically) include the possibility that there are *particular* nexuses in a world that arise and disappear (as in the emergence of natural laws). But if this coming into being and passing away would not

17 Of course, such a world would be inconsistent if understood in the normal sense of "world" – just as a game is inconsistent when one person is allowed to introduce the rule: "All people must wear blue hats", and, at the same time, another person may introduce the rule: "No person is allowed to wear any hat at all." But this only shows that such a "game" or "world" is not a game or world in any normal sense at all, where "normal" implies the norm of consistency.

stand – or, more precisely, would not occur – within *some* definite (determinate) nexus, then it would mean as much as the coming into being of a new world, or the passing away of a given world. But the determination (or objective definition) of a general nexus is simply a law or a rule.[18] It is therefore constitutive of worlds that they are determined by formal or axiomatic necessity. One quickly realizes that logic does not suffice to guarantee a definite nexus *within* a possible world. These nexuses, being particular, must be regulated in a particular way. In other words, logical connections alone are not sufficient for possible worlds to differ from one another. However, in order for worlds to be distinguished by nexuses *within* them, there must be something that regulates these nexuses and makes them comparable, so that one can then say: "This and that does not behave in this world as it does in that other world." But this thing that gives meaning to specific nexuses in a world is (specific) axiomatic necessity. A world *must* therefore be subject to certain nomological laws: these are constitutive for something to be a world. A world without such laws has no internal nexus whatsoever, and dissolves into indeterminacy.[19]

18 Space and time alone are not sufficient for the inner nexus of a world (i.e., insufficient to "hold a world together"). It is disputable whether we can individuate space and time at all, but if we assume that this is possible, then different worlds can be located within the same space and the same temporality (temporal extension), as long as things in different worlds have no effect on one another. Moreover, it is outright impossible to distinguish the purely abstract space and the purely abstract temporality of one world from that of another (i.e., to individuate them from each other). This is different in the case of a world with relativistic spacetime, as in the general theory of relativity, because relativistic spacetime can be locally curved, and its local curvatures may differ from those of the spacetimes of other worlds. But the questions of where and when and to what extent local curvature occurs in a specific world is only decided by the natural laws that prevail in that world. Hence, even in this case, we need natural laws beyond pure geometry in order to constitute the inner nexus of a world (and its difference from those of other worlds).

19 If this does not immediately make sense to someone, then the following illustration may perhaps help: At first sight, one may think that mere logical possibility suffices for there to be, e.g., a certain body with a certain mass in a certain possible world, and a certain other body with a different mass in the same world. But having mass is completely undetermined as long as it is not determined what difference it makes, e.g., what difference the relationship of different masses to one another makes within a world (e.g., that they attract one another, e.g., in proportion to the ratio of their masses, or something like that). But this is only determined by the

The Irreducibility of Inferential Necessity to Axiomatic Necessity

We have discussed why talking about formal necessity makes sense, and why it does not make sense to reduce modal linguistic practice to talking about material necessity. But now it may seem as if, *within* the field of formal necessity, inferential necessity can still be reduced to axiomatic necessity, that is, to the *axioms* of logic. The problem, however, is that these axioms also have to be applied in turn, and general principles are applied through inference. Let us say, e.g., that "$A \leftrightarrow \neg\neg A$" is an axiom of our propositional logic. If we want to apply this to "Socrates is a philosopher", then we must "see" that this expression can be substituted for "A" in that formula. Thus we implicitly "infer":

$A \leftrightarrow \neg\neg A$
"Socrates is a philosopher" can be substituted for "A"
$\therefore$ Socrates is a philosopher $\leftrightarrow$ Socrates is not not a philosopher.

Of course, we could formulate a rule that decrees in which case a linguistic expression can be substituted for "A": "Whenever a linguistic expression in English has the form xyz, it can be substituted for 'A' in '$A \leftrightarrow \neg\neg A$'." However, again, this rule would have to be applied. That such application is an additional step may become clearer from the fact that we can go wrong in this step. For simplicity, let us consider a mathematical example: Someone may take the rule "$x \times y = y \times x$", and apply it to "$5 \times 3 + 4$", in the following way: "$5 \times 3 + 4 = 3 + 4 \times 5$", on the grounds that $3 + 4 = 7$, and "7" can be substituted for "y". However, "$5 \times 3 + 4$" is evidently not a case of "$x \times y$".

General rules apply to singular cases. The application of rules cannot ultimately consist of applying a rule, namely the rule for applying rules, because then we would end up in an infinite regress of application

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special laws that affect mass. Logic by itself determines that what has mass does not have no mass (and similar general, formal nexuses). However, by itself, it is not sufficient to determine the particular differences that are made by "having mass". The mass laws required for this may be different in different worlds. But without *any* such laws in a world, having mass in the respective world is meaningless.

rules: there would have to be a rule for how the application rule for rules applies, etc.[20] Hence, since an axiomatic system, by definition, is based on general rules, it must presuppose the coerciveness of the application of these rules. It cannot incorporate this coerciveness into the system by incorporating general rules for the application of its general rules – at least it cannot do so completely. Hence, for any axiomatic system, there always "remains" an inferential coerciveness or inferential necessity which it cannot account for. Therefore inferential necessity cannot be reduced to axiomatic necessity.

We can also argue starting from Tarski's theorem: According to this, formal systems that are sufficiently explanatory (to express the basic arithmetic of natural numbers) are incapable of deciding or defining their own concept of provability.[21] If we metaphorically take the axioms of a formal theory to be certain places (e.g., in the space of discourse), and take proofs to be paths to other places (which stand for theorems), then this not only means that there are places to which the path network (of the respective axiomatic system) does not provide access. It also means that it is undecided or undefined (in the respective axiomatic system) what a path even is.[22] But this means that the coerciveness or necessity of the transition from one place to the other cannot (ultimately) be

20 That is why it does not help to locate the application rule in a metalanguage. For the metalanguage requires, in turn, metalinguistic rules (relative to itself). We would end up in an infinite regress of metalanguages. But that is not all: The application rule would concern the very relationship between the rule and the regulated. So, considered in terms of semantic ascent, it would concern precisely the relationship between the metalanguage and the object language, that is, it would regulate this relationship. Hence we would not only need further metalanguages, but also an additional language that is, so to speak, orthogonal to the semantic ascent, i.e., whose *object* is semantic ascent. But then we would need a language that regulates the relationship of *this* language to the relationship between the metalanguage and the object language, etc.

21 Cf. Beeh 2003, 97.

22 Admittedly, one can conceive of a "platonic" theory of metalogic, where Logic as such is given as a system (together with all its inferential paths) independently of any axiomatic system. In this case, this Logic beyond axiomatic systems would be the logic of inferential necessity – which is different from axiomatic necessity, i.e., the distinction I want to make would still hold. The difference from what I have suggested is only that, in this case, inferential necessity would not be dynamic, but, once again, systematically fixed (as a network of given paths).

a derived one (i.e., one that can itself be proven), and must therefore be original. (This is also immediately obvious: The definition or decidability of the concept of provability implies that if a proposition p is given, then it can be decided whether p is provable. But this would require a proof or, to use our visual metaphor, a path from p to its provability. But if provability defines what a "path" is, then the givenness of the path *to* provability already presupposes this provability – or else, it presupposes a *different* concept of provability, typically in a metalanguage – which in turn is undecided or undefined.)

The originality and irreducibility of inferential necessity is, so to speak, the positive flip side of Tarski's theorem: provability is undecided or undefined for sufficiently explanatory theories within those theories. However, it is *evident* that we do move from one proposition to another in the act of inference. (Again, it is not defined or definable what "evident" means, but we can at least *pragmatically* argue that Tarski's theorem is itself a result of a transition in the act of proving, that is, that it is at least evident to us that we (can) move from one proposition to another in the act of inferencing, insofar as we accept this theorem. And if we do not want to accept it, then we must *prove* that it does not hold.) Therefore this transition must *originally* (or "underivedly") be coercive (and not due to the prior givenness of a path), and this is precisely what inferential necessity is.

From a pragmatic perspective, this means that we must attribute to ourselves as subjects the ability to apply rules or concepts *originally*, that is, without the "how" of the application being defined in turn. We *implicitly* master the application of rules and concepts, and we (ultimately or originally) cannot master them in any other way. Nobody could have taught us how to apply rules at all: this must be part of our basic transcendental equipment (or "natural" equipment, depending on one's philosophical preferences), so to speak.[23]

For the *matter itself*, however, this means that we cannot avoid attributing original necessity to the (act of) application of principles or the (act of) derivation from them – that is, attributing to them a necessity that is not derived from principles – because that very necessity would be

23 Cf. I. Kant, KdrV B 172.

required for the derivation.[24] For this reason, the name "*necessitas consequentiae*" seems unsuitable for inferential necessity, because in philosophical usage it is not just used for the (pragmatic, operational) progression in logical reasoning, but also or even primarily for the *principle* of implication; that is, insofar as it is logically valid and hence necessarily valid. *Necessitas consequentiae* is given, we say, when the implication "A → B" represents a tautology, as in: "A → ¬¬A". We can, in fact, squeeze any transition to the conclusion in logical inference into the schema "Necessarily: A → B", for example: "*Necessarily: if:* ((if Peter is in Heidelberg, then he is not not in Heidelberg) and (Peter is in Heidelberg)), *then:* Peter is not not in Heidelberg"; where the entire expression in brackets after "*if:*" stands for "A". But by representing the transition in this way, namely as a necessary or "strict" or logical implication, we no longer represent it as the genuine coerciveness of the inference as such, but rather as a *principle* of this transition (or an application of this principle), which is abstracted from the process of inferring.

24 Interestingly, Wittgenstein formulates this idea clearly in the *Tractatus* (I omit the paragraph separations): "4.121 The sentence cannot represent the logical form; it reflects itself in it. What reflects *itself* in language cannot be represented. What expresses *itself* in language, we cannot express through language. The sentence shows the logical form of reality. It exhibits this form. 4.1212 What *can* be shown *cannot* be said. 6.522 However, there is the unspeakable. The unspeakable *shows* itself, it is the mystical." In my opinion, Wittgenstein slightly oversteps the mark with this: What reflects itself in a sentence, what a sentence exhibits, or what "shows itself" in it (in the sense that something shows "itself" in a mirror), is what *the sentence itself* cannot say. But *another* sentence can certainly talk about this and can at least try to say it. However, it is true that this other sentence must then already have a logical structure, which it can only show and not state, and so on *ad infinitum*. Hence the inevitability of inferential necessity remains, but it can certainly be made explicit, although never completely and definitively (see below). The logical structure of the world (and thus also of sentences, since they belong to the world as facts) is, in my opinion, nothing mystical – although that is of course the inevitable conclusion given the premises of the *Tractatus*.

Two Pragmatic Understandings of Logic

Inferential necessity is necessity *simpliciter*, in that we must always presuppose it in all our inferring and applying. Logic (as an enterprise) can be seen as an attempt to explicate this necessity-*simpliciter*.[25] However, this explicating is done by formulating this necessity axiomatically, because we cannot articulate it discursively otherwise.[26] This then often leads to a misunderstanding about what Logic (as an enterprise) actually represents. Because if Logic represents the explication of inferential necessity, then it is the explication not of an axiomatic system, but of what makes axiomatic systems possible in the first place, that is, what makes applications of and derivations from general principles possible (subjectively formulated: the foundation of the use of rules and concepts *as such*). We can therefore distinguish between two different operational or pragmatic understandings of Logic *qua* enterprise: Logic as an attempt to explicate inferential necessity; and Logic as an attempt to freely construct formal axiomatic systems that may then fulfill certain functions. The latter may also be called "logistics" (in contrast to the former) if you want. These two enterprises do not differ in the formal appearance of their results. We cannot tell from the sheer shape of a logical-axiomatic system whether it is

25 This, then, explains why it makes sense to search for *Logic* with a capital "L", and not just for one of many possible logics, that is, to construct coherent formal axiomatic systems. *Logic* is (or would be) the appropriate explication of the inferential necessity which is assumed in all application and deduction. (Cf. Wittgenstein, *Tractatus* 6.1223: "It now becomes clear why one often felt as if the 'logical truths' had to be *'requested'* by us: namely, we can request them insofar as we can request a sufficient notation.") However, it may (or must) be the case that this explication will always remain imprecise and incomplete, which is why there is definitely a certain scope for ambiguity and therefore also for plurality in this undertaking. There may be, on some points, a dispute about what the correct Logic is, and in some cases, there may be no directly obvious solution to that dispute. In principle, however, this "correct" logic is what is called "classical logic".

26 Logic in general articulates the original synthetic unity (which is, at the same time, *originally synthetic*) of inferring (as an original act). Formal, axiomatic logic analytically explicates this unity. (This was already stated by Kant, KdrV B 134, annotation, albeit within the framework of his transcendental idealism which, in my opinion, is not theoretically sustainable.) However, the analytic explication of synthetic unity necessarily always remains incomplete (or, alternatively, gives rise to inconsistencies).

intended to represent a free construction or the explication of inferential necessity. But the difference in comprehension between the two undertakings is fundamental. In particular, the latter project is not allowed to assume from the outset (1) that inferential necessity can be adequately represented axiomatically at all, or (2) that such an explication can be complete and/or consistent, or (3) that it can be clearly decided which of the – possibly – different explications is the "correct" one, i.e., that one formal logical-axiomatic system could be distinguished from all the others according to internal, formal criteria. But this means that, conversely, if it turns out that all of this or at least some of this should *not* be the case, then that in no way speaks against inferential necessity or against its determinacy, consistency and coerciveness.

That is, the fact that axiomatic systems can only represent attempts to explicate inferential necessity, but not the latter itself, explains why there can be incompleteness and uncertainty in this explication and why there may be different ways of explicating it. Finally, this also explains the plurality of logics that emerged in the 20th century: If Logic is just understood as an axiomatic system and no longer as an explication of a necessity that underlies all axiomatics, then we can of course design "logical" systems as we wish – as long as they are consistent and reasonably complete. (However, in order to apply these two criteria, we must in turn assume inferential necessity.) There may be a hierarchy of axiomatic systems, e.g., a logical axiomatic system may be the basis of a nomological axiomatic system. But this hierarchical priority *within* the field of axiomatic necessity is only a relative one; only a priority within the same type of necessity, namely the axiomatic one. *Inferential* necessity, on the other hand, categorically conditions axiomatic necessity; it is presupposed by it *as such*.

Nonlogical Axiomatic Necessity

Axiomatic necessity – that is, the "authentic" one, which does not represent the explication of inferential necessity – is, in contrast to inferential necessity, not a necessity-*simpliciter*, but a particular one – or better: one that introduces particular necessity into reality (or into logical space). Because, as we have seen, the laws of nature, and even more so other laws

such as the rules of chess, are not necessary *simpliciter* – it is easily conceivable that counterfactual natural laws prevail in a (logically possible) world different from ours. However, it can be observed that philosophers, especially modal logicians, are reluctant to deal with such particular nexuses of necessity and prefer to stick to logical necessity (which they generally understand as axiomatic). But in our everyday speech we rarely talk about logical necessity; we are mostly interested in necessities and possibilities that are given under particular laws, be these the laws of nature or others. Furthermore, as I have said, the fundamental difference does not lie *within* the field of axiomatic necessity, between logical-axiomatic and non-logical-axiomatic necessity, because this difference could only be a relative one. The character of the necessity of logic as *simpliciter* or absolute, and thus its difference from the particular necessity of laws of nature and other nexuses, lies in the fact that the former is (originally, in itself) not an axiomatic necessity, but *inferential* necessity, which underlies the axiomatic necessity as a condition of possibility. *Within* the field of axiomatic necessity considered on its own, there is no reason to "absolutely" privilege a particular axiomatic system by attributing necessity in the material sense to this one system alone. (This is quite unavoidably the case and therefore also applies to Logic, *if* Logic is taken to be a purely axiomatic system. Then logical pluralism is trivially correct.) For even if this one system is considered fundamental to the others, its necessity is not fundamentally different from that of the other axiomatic systems. Only the contrast between inferential and axiomatic necessity can manifest this fundamental difference. But as soon as we make the former explicit (as "logical" axiomatic necessity), we inevitably transfer it into the field of axiomatic systems and thereby relativize it.

There is therefore no good reason – contrary to the reduction of inferential necessity to axiomatic necessity considered above – to limit modal speech within the field of formal necessity to inferential necessity and to withhold the title of "necessity" from axiomatic necessity. This can also be seen from the fact that limiting our modal speech solely to logical necessity would greatly impoverish our (possible) knowledge, because with regard to the future, and to a large extent also with regard to the past, we can only gain knowledge through inferences on the basis of particular *axiomatic* necessity – as when we say that when an atom splits, it

will necessarily emit radiation, that is, due to its essential properties and the laws of nature.

The Necessity of Theorems

If we combine axiomatic and inferential necessity, then we arrive at the necessity of the theorems that can be derived from the axioms – regardless of whether these axioms are logical ones, namely attempts to axiomatically articulate inferential necessity, or axioms of a different type, such as nomological ones. The necessity of theorems follows directly from inferential and axiomatic necessity, that is, it also represents *formal* necessity. Hence, if one wishes, one can count this as a third type of formal necessity; although since this type follows directly from the first two, it is not of much interest.

In formal logic, this necessity is expressed by the aforementioned Necessitation Rule in its narrower sense: If p is a theorem, then *necessarily-p* is also a theorem. It is crucial for modal theory that the necessity of its theorems does not have to be based on material necessity, i.e., on the content of possible worlds, but can simply be decided formally. To put it colloquially: We can simply test whether p is a theorem without considering the question of modality, *and then*, if the result is positive, assign necessity to p. This necessity then (primarily) indicates simply that p is a theorem, and nothing more – just as the axiomatic necessity of q (primarily) indicates nothing more than that q is an axiom – albeit *in view of* material necessity (see above). We can, so to speak, take theorems with us from nonmodal logic into modal logic (that is, by attributing necessity to them). For our discussion this will mean: We can *without further ado* (*or further input*) take theorems (as well as axioms) from the field of formal necessity over to the field of material necessity; they "automatically" precipitate materially.

This means, however, that *formal* necessity, including the necessity of theorems, is unoriginal (not genuine) insofar as, viewed with regard to the sphere of the formal itself, it just says that the axioms are axioms and the theorems are theorems. In the field of the formal itself, this state of affairs has no modal meaning whatsoever. (In other words, the modal

formulation contributes nothing to the axioms and theorems in this field. It does not make them "even more irrefutable" or anything else. The modal formulation of the validity of axioms and theorems makes sense solely in view of material necessity.) This also applies to the axioms and theorems of *modal* logic: they speak of modality in terms of content, but as such, *qua* axioms and theorems, they are not *necessarily* given, but are simply *given*. As I have said, axioms and theorems get their modal meaning from the Necessitation Rule. Specifically, this means that if a "space of possibilities" is opened up (in thought or in reality) under certain axioms, then these axioms and the theorems derived from them are reflected in this space of possibilities as necessarily given *within* this space. The Necessitation Rule, so to speak, articulates this reflection or this "precipitation" or substantiation of formal necessity as material. Since a space of possibilities that is not subject to *any* axiomatic determinations is completely inarticulate and therefore decrepit – that is, since a space of possibilities can only be sensibly opened up (or can itself open up) under an axiomatic system, the Necessitation Rule is constitutive of modality. On the other hand, in a certain way, this weakens its meaning. This rule does not award the axioms and theorems anything that they would not have had without it, so that modality would be (logically) prior to axioms and theorems and *then* attributed to them, in a (logically) secondary step. No, modality (in the genuine sense) only opens up under axioms, and the Necessitation Rule, so to speak, only expresses the right that the respective axioms and theorems have from the outset over the respective space of possibilities, precisely because this space has been constituted under their specifications. But this means that formal necessity takes its meaning from material necessity. If there were no material necessity, then it would be meaningless to say that, for example, logic is *necessarily* valid.

This has consequences for the theoretical possibilities for ontology, which I want to outline briefly. The respective space of possibilities and, consequently, the corresponding modal ontology, do not have to answer for modeling the specific necessity of the axioms and theorems. We do not have to make any additional ontological arrangements for axioms and theorems to be reflected in material necessity. It suffices that there is *formal* (i.e., immaterial, i.e., ontologically speaking, *nonentitative*) necessity, that is, inferential and axiomatic necessity. Specifically, this means that, with regard to the latter, it suffices (ontologically speaking) that there are,

for example, natural laws. We do not have to introduce any other entities into our ontology besides these (such as special structures of the world, or merely possible worlds or things) in order to ontologically guarantee the necessity of theorems.

Specifically, this means that the logic of *formal* necessity is the so-called modal logical system "S5"[27], that is, the maximal normal modal logic that is not identical to predicate logic (i.e., in which *p* is not equivalent to necessarily-*p*). But formal necessity is independent of further ontological conditions (apart from the validity of the axioms themselves), i.e., for example, it does not need a system of possible worlds in order for it to be given. Therefore it is not ontologically necessary to introduce such a system of possible worlds (or any other equivalent ontological conditions). In order for there to be inferential and axiomatic necessity, it is *not* necessary to develop an ontological model that corresponds to the logic of this necessity (i.e., the system S5). It is therefore not necessary for the logical space of possible worlds to exist in an ontological sense. Of course, we can develop corresponding *semantic* models to articulate formal necessity materially. But we need not burden ontology with such models. (Consequently, in my opinion, we should not do so. But this would be a different discussion.)

Modal pragmatics therefore reveals a negative or "unlimiting" determination of the modal ontologies that are theoretically compatible with it: such modal ontologies do not have to accept all logically possible worlds as real. As I have said, this is completely compatible with the fact that the logic of formal necessity is adequately described by the system S5 – to which (logically), in possible world semantics, quantification over all logically possible worlds corresponds. It is therefore conceivable that a modal ontology should only accept a specific selection of possible worlds as real – e.g., the nomologically possible worlds. And finally, it is theoretically possible (i.e., it is compatible with formal necessity) to conceive of a model of modal ontology in which only *one* possible world is real, namely the actual one, but which, e.g., expands modally into the future. This could be completely sufficient to ensure that possibility and necessity can be spoken of meaningfully and truthfully in reality – including

27 Cf. e.g. Uwe Meixner 2008, 20–26.

the insurance that *axiomatic* and *inferential* possibility and necessity can be spoken of meaningfully and truthfully. Our talk of logical possibility does not require logical space (including all the possible worlds in it) to be ontologically given; for it is possible to design an ontology which, under its modal aspect, is limited to just a "few" or even just one real world (in the broadest sense), and which nevertheless supports meaningful and truthful talk of logical necessity, and of formal necessity in general. Obviously, I emphasize this point because I myself defend such an ontology. However, of course, modal pragmatics does not commit us to such an ontology; nor does it even clearly suggest it. It only opens up the theoretical possibility, the possibility of a sufficiently clear, distinct and consistent formulation of such an ontology.

2. Material Necessity

Formal necessity, both inferential and axiomatic, is not only indispensable; it is in a certain sense even more fundamental than material necessity – that is, from a formal point of view. (Its priority is a tautology.) Nevertheless, it is material necessity that determines our "normal" understanding of necessity. We could say that we presuppose formal necessity *for the sake of material necessity*: typically, we want to talk about material necessity, and to do this we presuppose formal necessity. Material necessity occurs in our colloquial speech, again, in two forms, whose distinction, in a certain sense, runs parallel to the distinction between the two formal necessities as *simpliciter* and *particular* – that is, in the forms of unconditional necessity and conditioned necessity.

2.1 Unconditional Material Necessity

Sometimes we use the word "necessary" to express the unconditional necessity of an intramundane state of affairs. For example, we say: "All human beings must die." Expressed in PWS, we typically want to say that, necessarily, in all nomologically possible worlds, all existing human beings die at some point.

As I have said, unconditional material necessity can or must *also* be the necessity of the validity of the axioms and theorems, in the sense of §1.2, *in* those worlds that have been selected under these axioms, e.g., the validity of the laws of nature in all nomologically possible worlds. For, without this intramundane necessity of the respective axioms, no material inferences could be made from them. In this case, unconditional material necessity is simply the mundanization or materialization of formal necessity – that is, according to the Necessitation Rule (in its broader sense) – and does not go beyond this. (The necessity of analytic sentences such as "All bachelors are unmarried" is also a formal one in the sense explained.) *Such* material necessity is "unfounded" or "simply exists": there is no reason why a particular natural law or logical axiom is necessary, i.e., why it is valid in all (nomologically) possible worlds. The "reason" that, e.g., the laws of nature are necessary with regard to

our world is simply that they are valid in our world, or that our world is a nomologically possible world. But this explanation is of course tautological, because the fact that something is nomologically necessary means that it is the case in all nomologically possible worlds. To put it another way, if we accept the materialization of formal necessity – which seems inevitable – then we accept unfounded necessity.[28] Unfoundedness in this sense is something different from unconditionedness – there can (possibly, see below) not only be unfounded, but also well-founded unconditioned material necessity. *A fortiori*, unfoundedness is different from inferential necessity's characteristic of "being *simpliciter*", because materializations of natural laws are also unfoundedly unconditionally materially necessary.

In a next step, theorems in the broader sense – let us call them theorems* – may be unconditionally materially necessary (under certain circumstances), if they exist. Theorems in the narrower sense are those that are derived from a single axiomatic system, while theorems* are derived from the combination of two mutually independent axiomatic systems. A clear example of the latter is the combination of nomological and moral modality. For example, if I ask myself what I must do (in a moral sense) in view of the fact that I see a person drowning, then it is mandatory that I

28 This is interesting because many philosophers believe that unfounded necessity is unacceptable, at least when it comes to the necessary properties of individuals, such as an electron's property of having a certain electrical charge. Some believe that this property can only necessarily be possessed by an electron if it has another property *on the basis of which* it accrues to the electron – e.g., the property of having a certain essence. We can immediately see that this leads to an infinite regress, which is why some people think that there are no properties that *objectively* necessarily accrue to something, but only properties that necessarily apply to something on the basis of how it is described – e.g., as an electron (which then necessarily has an electrical charge), or as a bachelor (who then necessarily is unmarried). However, if we assume, in consequence of what has been presented, that there are cases of unfounded material necessity anyway – namely of materialized formal necessity – then the general argumentative situation indicates that we should demand from those who *reject* unfounded essential properties a justification for why such properties should *not* exist, rather than demanding from the essentialist a justification for why there *may* be unfounded necessity with regard to particular entities. If there are essential properties, then they must ultimately accrue to things in the same way that the laws of nature accrue to our world (if we assume that they accrue to it), namely simply or unfoundedly.

combine the moral requirement of rescue with the nomological requirements of this rescue. Only then do I arrive at the necessity of (e.g.): "I must jump into the water and pull out the drowning person." In a universe with different natural laws, I would perhaps arrive at the conclusion: "I must use my telekinetic powers to levitate the drowning person's body." Of course, the question is whether *in ontology* there (possibly) are axiomatic systems that are independent of one another (i.e., not hierarchically dependent on one another) and which can simultaneously apply to a single world – but at this point we are only exploring theoretical alternatives. For example, we could – purely theoretically – assume that the behavior of physical objects towards one another is regulated by "relational natural law", while the essential properties of such objects are determined by a different kind of law. If we also assume that these two types of laws are independent of each other (at least to a certain degree), that is, that certain (sets of) essential properties are compatible with different (sets of) natural laws[29] and vice versa, then it is only the concrete world (in question) that connects the two and therefore enables the derivation of "mixed" theorems*. In our above example of the necessity of human mortality, we assume that certain *natural laws* are valid, and that human beings have certain *essential properties*, such as having a certain organic form. If these two assumptions were axiomatically independent of each other, then it would be a theorem* that all human beings are mortal. Theorems* are therefore – if they exist – world-dependent, and therefore not *formally necessary* (they are only formally possible).[30] If such theorems* were ontologically valid, then, by this alone, unconditional material necessity would have an *ontological* meaning that goes beyond the materialization of formal necessity.

29 For example, one type of law determines which particles have how much mass, while the other determines how strongly they attract each other, depending on their amount of mass and their distance from each other (or possibly from something else). It would then be conceivable that the same types of particles (with the same respective amounts of mass) in our actual world will be subject to different natural laws in other possible worlds.

30 For example, we could imagine that human beings (while having the same essential properties as in the actual world) do not necessarily ever die in a possible world with different natural laws – e.g., a world where time does not proceed linearly, but runs in a circle.

Nevertheless, we usually talk of such unconditional material necessity, which represents a materialization, namely a necessitation of formal necessity, because modal statements that concern individuals are typically articulated within the framework of conditioned material necessity, as discussed below. However, "materialized" necessity appears not only in assertions, but also, for example, in assumptions. For example, someone observes that every time water is observed flowing, it flows downhill. They now formulate the assumption that this does not just happen by chance every time, but that water, when liquid, *necessarily* flows downhill. The possibility of formulating such assumptions is important, among other reasons, because they can be the starting point for scientific investigations. In the case of an assumption, the premises are typically not yet known (e.g., the law of gravity and the essential properties of water). Hence the assumption *simply* expresses the unconditional materially necessary state of affairs that, in our universe, water flows downhill – albeit only in the form of an assumption. However, this shows that material unconditional necessity *originally* occurs in our ordinary speech practice (albeit often only in the form of assumptions), and not only as a conclusion drawn from premises of an axiomatic character. Unconditional material necessity is therefore fundamentally different from formal necessity.

Furthermore, unconditional material necessity is necessity *in the matter itself*, and not just the necessity of the conclusion in the sense of inferential necessity. The sentence "All human beings must die at some point" obviously has a different modal meaning than the sentence "The gardener must have been the murderer". The difference between unconditioned (material) necessity and (formal) necessity-*simpliciter* also is evident from the fact that unconditioned material necessity can also occur in the context of necessity that is non-*simpliciter*, i.e., in the context of some particular axiomatic necessity (i.e., of such axiomatic necessity that is not an axiomatization of inferential necessity). This happens, e.g., in the context of the natural sciences, with regard to the laws of nature, e.g., in the form of the unconditional necessity that all human beings must die – in all nomologically possible worlds, but not in all logically possible worlds. *Conditioned* necessity, which will be discussed below, is, on the other hand, *intramundanely* (i.e., materially) conditioned, and not only formally restricted (e.g., to nomologically possible worlds).

Modal-pragmatic Meaning is Not Always Reflected in Linguistic Form

Before we move on to conditioned material necessity, it seems useful to briefly emphasize the character of the systematization undertaken here. As I have said, inferential necessity can be represented axiomatically – in fact, only through axiomatization can it be represented, that is, made explicit. Originally, this necessity is implicit in inferring. Axiomatic necessity, in turn, can or must appear within the world as unconditional material necessity. This of course means that inferential necessity *as axiomatized* also appears inner-worldly as unconditional material necessity.[31] Hence we cannot tell from a sentence that articulates logical necessity whether it is talking about inferential, axiomatic, or unconditional material necessity; and we cannot tell from a sentence that articulates nomological necessity whether it is talking about axiomatic necessity or (unconditional) material necessity – for these determinations concern a *pragmatic* linguistic differentiation of sentences. Only sentences that articulate such unconditional material necessity that has no axiomatic character, such as "It is impossible that Peter is not identical with himself", clearly speak only about unconditional material necessity, and not about necessity in the sense of §1.1 or §1.2.

The operational or pragmatic differentiation of our talk of modality developed here is, so to speak, orthogonal not only to modal logic and

31 It should be noted here, without going any deeper, that (as already indicated) the logic of this materialized axiomatized inferential necessity is trivially the strongest possible modal logic, namely the one described by the modal logical system called "S5" (cf. Meixner 2008, 20–40). This system differs from nonmodal predicative logic only in that the axiom "$\Box A \leftrightarrow A$" does not apply. However, this axiom applies to all logical tautologies – this is exactly what the materialization of inferential necessity says, which from a formal point of view represents the so-called Necessitation Rule: If "p" is derivable, then "$\Box$p" is also derivable. However, it follows from this that the logic of materialized axiomatized inferential necessity is the one that differs from simple predicative logic only in that the axiom "$\Box A \leftrightarrow A$" is valid only restrictedly, namely in the form of the Necessitation Rule. Conversely, this already indicates that if modal *ontology* is supposed to be richer or stronger than what results from the materialization of inferential necessity alone, i.e., if (potentially) something else is necessary, other than just logical states of affairs, then the corresponding modal *logic* must be weaker, that is, it must be based on fewer axioms than S5.

semantics, but also to the classification of modality as logical, metaphysical, nomological or something else of this kind. Logical necessity, for example, can appear both as inferential and as axiomatic necessity, and furthermore as unconditional material necessity, even though its meaning stems intuitively from inferential necessity. Conversely, this means that when we consider unconditional material necessity, we find the other types of necessity discussed so far gathered together in it, so to speak: the laws of nature and the laws of logic are *also* given within the world, not only formally. In this sense we can say that unconditional material necessity unites within itself (or comprises) the two formal types of necessity. However, this does not mean that the latter are *reducible* to the former (see above). On the contrary, inferential and axiomatic necessity are formally *presupposed* by material necessity.

There is therefore a relationship between formal and material necessity that is similar to that between transcendental forms and empirical objects in Immanuel Kant's philosophy. The transcendental forms are presupposed by empirical objects as conditions of their possibility. But being a presupposition no longer means absolute priority within the framework of Kantian theory, as was typically the case in earlier traditional metaphysics, because the transcendental forms are "suitable" exclusively for the formation of empirical objects. Without empirical objects they are meaningless; they have no purpose at all. Therefore – in this sense – the transcendental forms *depend* on that whose prerequisite they are, although of course not in the same way in which the latter depends on them as conditions of possibility. However, empirical objects, or the empirical world, have priority over the transcendental forms in that they not only give them meaning, but also *unite* them within themselves together with the material of intuition. The transcendental forms are *also* or even *foremost* encountered in the empirical world, and (epistemically speaking) only secondarily in transcendental reflection. (It is from here that – speaking in the context of Kantian theory – we get the impression that, e.g., causality is a rule which the world prescribes to our understanding, and not the other way around, as is actually the case according to Kant.) Similarly, material necessity unites within itself inferential and axiomatic necessity, which are presupposed by it as conditions of possibility (albeit not only – as in Kant – with regard to our cognition, but as such).

2.2 Conditioned Material Necessity

We often use the word "necessary" to express *conditioned material necessity*. This necessity is given when something is not unconditionally necessary, i.e., does not exist in all possible worlds (be this in all logically possible worlds, or in all worlds subject to a particular legislation, such as the nomologically possible worlds), but only in all those possible worlds that fulfill a certain *intramundane* condition. *Actually* conditioned material necessity is given when the actual world is among these possible worlds. Of course, we typically talk about this because we are usually interested in what is (conditionally) necessary or possible in *our* world.

More precisely, actually conditioned necessity is given when, first, there (actually) is a formally necessary nexus, for example that it is necessary that if a person is a bachelor, then that person is unmarried. Secondly, if it is true that Peter is *actually* a bachelor, then it cannot be concluded that it is unconditionally necessary that he is unmarried – he could also have married. But it is then *conditionedly necessary* that he is unmarried: in all possible worlds in which the condition that he is a bachelor is met, Peter is unmarried.

In our talk of conditioned material necessity, we substitute within the axiom schema K, "$\Box(A \rightarrow B) \rightarrow (\Box A \rightarrow \Box B)$", the simply given necessity of A in the second pair of brackets by "posited necessity" or "posited irrevocability", or, in PWS, by a posited selection of possible worlds – that is, according to an intramundane criterion.

Talk of conditional necessity makes sense *prima facie* because it marks or can mark a difference in the matter, or in reality (in the broadest sense). For example, in the actual world, it may be the case that all residents of the house at Krämergasse 2 in Heidelberg are unmarried (at all times). Now, if Peter is *actually* a resident of this house, then it follows that he is unmarried. But it does not follow that *necessarily*, he is unmarried,[32] because there are of course possible worlds in which some resident of the house at Krämergasse 2 in Heidelberg is married (that is, in this case it is not true that $\Box(A \rightarrow B)$, but only that $A \rightarrow B$). On the other hand, it is true that, (conditionedly) necessarily, Peter is unmarried if he

32 In the sense of material necessity, that is. It would only be true if the respective necessity were understood as *inferential* necessity.

is *actually* a bachelor, because there is no possible world in which any bachelor is unmarried. Conditioned necessity denotes a real distinction as compared to mere actuality – a distinction that is *given* under the conditions we assumed. There are states of affairs that are characterized by the fact that, although they are not unconditionally necessary, they are nevertheless necessary under a certain condition. These states of affairs differ from those that *are* given under a certain condition but are not *necessarily* given under that condition – as in the example of Peter as a resident of the house at Krämergasse 2, Heidelberg.

Conditioned necessity is not identical to inferential necessity, because in the latter sense we can also say that, necessarily, Peter is unmarried since he is a resident of the house at Krämergasse 2 in Heidelberg (where all residents of this house are unmarried – contingently, not necessarily).

However, in order to distinguish conditioned necessity from inferential necessity *in every case*, we must remember a condition that I formulated at the end of my explanation of the latter: If, in defining the criteria for the selection of possible worlds, *all* material conditions of the inference in question are taken into account (in the above example: not only the premise that Peter is a resident of the house at Krämergasse 2 in Heidelberg, but also the premise that all residents of this house are unmarried), then the necessity, which is then formulated by quantifying over these selected worlds, is trivial and therefore meaningless. Accordingly, we must now say that if the state of affairs whose necessity is to be represented is *simply identical* to the state of affairs that is set as a condition, then we are no longer dealing with conditioned necessity, but only with inferential necessity. The sentence "If Peter is a bachelor, then it is necessary that he is a bachelor" no longer expresses conditioned material necessity, but only the inferential necessity of concluding A from A.

It follows from this that there is a certain degree of discretion or arbitrariness as to what is to be regarded as a case of conditioned material necessity and what is not. Intuitively, the states of affairs that Peter is a bachelor and that Peter is unmarried seem to be sufficiently different from each other to allow the above example to be understood in terms of conditioned necessity. (After all, we need more than just bare formal logic to infer the latter from the former; that is, we need to understand the concept of a bachelor – whereas in concluding A from A we do not need to understand the particular conception of A, but only need to be

able to determine that A and A are the same conception.) But, of course, one can also be stricter and regard the talk of necessity in this example as a case of inferential necessity. Unequivocal examples of conditioned material necessity are those where the first premise does not formulate a nexus which is conceptual-analytic, but instead is nonanalytic and axiomatic. For example, if Peter is currently in free fall above the earth, we can say: "*Conditionedly necessarily*, Peter is accelerating at a rate of approximately 9.81 m/s^2." This acceleration does not result merely from the fact that Peter is in free fall above the earth (which is not necessary, but only actual) – both he and the earth having the specific mass that they currently have – but only from this together with the specific law of gravity that is in force in our universe.

This clarifies once again that the *condition* in the sense intended here is something different from the particular axiomatic context of modal speech. The latter concerns modality itself, that is, the *way of* being necessary, possible, impossible or nonnecessary. The condition, on the other hand, does not typically stand (within the respective inference) as a *modal* state of affairs, but merely as an *actual* one. Of course, this state of affairs may also have modal significance from other points of view. According to our normal understanding, what is actual is also possible. And the actual state of affairs may in turn be conditionedly necessary due to the existence of other conditions. But in the respective inference the condition stands not as modally given, but merely as *actually* given. The question of conditioned possibility is therefore – from the outset – indifferent to the plurality of modal contexts. For example, if it is *morally* necessary to help other people whom we see in situations of emergency from which we can help them escape, and Mary sees Peter in such a situation, then it is morally *conditionedly necessary* that Maria helps Peter.

We must therefore distinguish between *axiomatically determined* modality and *materially conditioned* modality. Nevertheless, there is a parallelism between the two in that, expressed in PWS, both concern a limited selection of possible worlds from all the possible (*simpliciter*) worlds. In the case of axiomatically determined modality, this selection is formal, or "made from the outset": If, e.g., we talk about nomological necessity, then only nomologically possible worlds are taken into account from the outset. In the case of conditioned material modality, on the other hand, the selection is made *ex post*, *within* a given modal

framework, e.g., an axiomatically determined selection of possible worlds: a selection is made according to the criterion of the existence of certain nonmodal states of affairs *in* the respective worlds. Nevertheless, viewed abstractly, in both cases we are dealing with a limitation of the set of possible worlds. Therefore, from this abstract point of view, there is no reason why the first form of this selection should be permissible or "meaningful" while the second should not.

At least in the context of PWS it seems obvious that there is no reason to prohibit talk of conditioned necessity. For the corresponding sets of worlds that it resorts to are, in fact, given: For example, there is *de facto* the set of all the possible worlds in which Peter is a bachelor. Besides, as just explained, the procedure of using a limited selection of possible worlds as a basis for talking about certain modal contexts is a completely normal one, without which we (in the context of PWS) would not be able to talk about nomological necessity, for example. Hence there is no reason why, in inferences based on specific necessity, we should be permitted to limit the selection of worlds only with regard to lawlike premises (e.g., to all nomologically possible worlds), and not also with regard to premises that express concrete states of affairs (e.g., that Peter is in free fall above the earth).

Now, conditioned necessity is *nontrivial* in the same sense in which, e.g., the (material) necessity of the validity of the laws of nature in all nomologically possible worlds is *trivial*. It is also *this* necessity that we talk about in most cases when we talk about necessity in everyday life, and what we usually ask about when we want to know whether something must be or can be that way. When I am on a visit out of town and I ask my host whether I can still catch the train at 7 p.m., I do not want to know whether there is a possible world in which I catch it, or even whether there is a nomologically possible world in which I catch it. I want to know whether I can catch it *under the given conditions*, or whether I will necessarily miss it. I will be somewhat confused, not to say annoyed, if, after I run to the station and discover that the train left 5 minutes ago, my host tells me that they had affirmed this possibility because there are, without doubt, possible worlds, even nomologically possible worlds, in which I catch the train.

From this latter example we can already see that *positing* or *setting* a condition *qua* posited necessity is not always subjective in the sense of being related to how we speak of something. (If we speak of Peter as a bachelor, then he is necessarily unmarried. If we speak of him as a civil engineer, then he may be married.) A posited necessity can also be objectively posited. Such objectively posited necessity is colloquially called "irreversibility", which already suggests that it has to do with temporal conditions (more on that below). In our example, the conditions posited include the time of the question, the location of the questioner (including their distance from the train station), their maximum possible speed of movement, and much more. All of these conditions are objectively irreversible at the moment the question is asked – they have nothing to do with how the questioner speaks of themself (in fact, the questioner need not speak of themself in any specific way at all).

The Indispensability of Conditioned Material Necessity

There is no need to argue for the indispensability of *unconditional* material necessity, because it intuitively represents the paradigmatic case of real necessity (albeit not the most frequent case in our ordinary linguistic practice). In the case of conditioned material necessity, its indispensability is perhaps not immediately obvious. First of all, it should be reiterated that speaking of it is meaningfully *possible* within PWS, because PWS provides everything that is needed to establish this speech practice. But why should we speak of conditioned material necessity within the context of modal theory, and not just leave it for our everyday talk of trains yet to be caught and the like?

Without the concept of conditional necessity, we could not speak of the necessity of states of affairs that directly involve the existence of individuals in space and time (including individuals in a broader sense, such as the rain or the raindrops that exist when we truly say: "It is raining"). Because, with very few, very questionable exceptions (e.g., God's individual existence, which according to some people is necessary, although it is questionable whether this should be thought of in spatiotemporal

terms), such existence is not necessary – or at least that is the intuition of most people.[33] Only universal nexuses of concrete things can be materially *unconditionally* necessary, such as the fact that it is necessary that I will always miss my train if I set off on foot from my host's house less than 15 minutes before its departure from the train station.[34] In all nomologically possible worlds, whenever I set off on foot from my host's house to the station more than 15 minutes before my train leaves, I will miss that train. (Assuming my host's house is at a certain distance from the station and my movement on foot cannot exceed a certain speed, etc.) On the other hand, it is not *materially unconditionally* necessary that I will miss this train, *now that I in fact* set off on foot from my host's house less than 15 minutes before my train's departure. Because this is not the case in *all* nomologically possible worlds, but only in all those in which I leave less than 15 minutes before my train leaves, i.e., in which this additional specific intramudane condition is satisfied. So, in the sense of unconditional necessity, my host would be right if they told me that I did not *necessarily* miss the train. I just missed it (in this sense) as a matter of fact. However, *in fact*, I can also miss the train *accidentally*, i.e., even if it is possible (but not necessary) for me to catch it – and this means that I could have caught it. Without the concept of conditioned material necessity, we cannot differentiate *concrete singular* states of affairs modally.

Now, at first glance, the question of concrete singular modal states of affairs does not seem to be very important for modal theory. But in theory, significance is not always apparent at first glance. It often only becomes evident when we think things through. If we think through a modal theory in which there are *no* concrete modal states of affairs, we notice that in such a theory it cannot hold that in the above example (the actually existing) Peter *necessarily* accelerates at 9.81 m/s^2. It may well be necessary that Peter, *if he is in free fall*, accelerates at 9.81 m/s^2. But without conditioned necessity, it can only follow (under the additional premise that Peter is in free fall) that he *actually* accelerates at 9.81 m/s^2, and not that he *necessarily* accelerates at 9.81 m/s^2 – at least not if we assume that Peter does not necessarily exist (and is not necessarily

33 In contrast, e.g., to Timothy Williams, 2013.

34 Of course, the existence of abstract entities such as numbers is also unconditionally necessary, if we accept them as entities.

in free fall – as would in fact be the case in a completely deterministic universe). In such a modal theory it could very well be the case that, *necessarily, everything which is in free fall accelerates at 9.81 m/s²*, but this general state of affairs can never become singular and concrete (at least not with regard to contingent entities): it cannot be the case within this framework that, *necessarily, Peter* (*who exists*) *accelerates at 9.81 m/s²*, because Peter does not necessarily exist.

One might be willing to live with that. But if we can no longer modally differentiate the concrete states of affairs in a world, then we can no longer concretely bring to bear any specific axiomatic necessity, particularly any nomological necessity. As already noted above, a distinction must be made between worlds in which a certain natural law is in force and those in which no violations of that law occur, because the latter may also be the case by chance, without that law being in force. But if there are no concrete modal states of affairs, then it can no longer be differentiated objectively, i.e., according to the subject matter, whether a (concrete) apple simply falls from the tree to the ground by chance, or whether it does so out of necessity because, e.g., the law of gravity is in force in that world. It may still be the case that the law of gravity *is* in force in the world in question. But this does not result in any *necessity for* any particular thing in this world, because the inference to this necessity is not permitted. But this means that the laws of nature do not change the concrete constitution of the world. If there are two worlds in which there are no violations of a certain natural law, but in one world this law is valid and in the other it is not, then the only difference between these two worlds is that in one world the law is in force and in the other it is not. (Except that, strictly speaking, these two such worlds will also differ in all *general derivations* from the natural law for individuals, such as: "Necessarily, whenever Peter has mass, he attracts other bodies.") But in all *concrete* states of affairs – that is, in all states of affairs that involve individuals in space and time in actuality (and not just hypothetically, as in the example just mentioned) – this law makes no difference between the two worlds. They are exactly the same in terms of their concrete states of affairs – in our above example, the apple falls *merely* as a matter of fact in both worlds; in neither of them is the concrete state of affairs of its falling necessary. The laws of nature never "reach" concrete reality, so to speak. It is true that in a world in which certain natural laws are in force, no violations of these

laws will ever occur. But as I have said, the laws of nature are not required for this. There are possible worlds without them (i.e., without these laws being in force) in which no violations of these laws actually occur. But then the laws of nature would lead to a completely unnecessary duplication of possible worlds, because for every possible world with certain laws of nature there would then be a duplicate without these laws of nature, which is exactly the same as the former in everything except that these laws are valid in it (and the general derivations from these laws are also valid). But then it seems superfluous that there should be any laws of nature at all. If, on the other hand, there is conditioned necessity, then the two worlds above differ in their *concrete modal states of affairs*, e.g., in one of them a certain, concrete apple does not just fall to the ground in fact, but also does so (conditionedly) *necessarily*.

Hence, if there were no conditioned necessity, then it would follow that the observation of concrete states of affairs would *in principle* never entitle us to assume the validity of natural laws, but only to formulate general propositions regarding our actual world – and even this only to the extent that we have observed this world so far. Because *in reality* we would never deal with necessary connections (we would never be objectively related to such connections), but only factual ones. (What is in question here is not just the epistemic problem of induction, but a more fundamental problem of reality itself.)[35] We would never be objectively entitled to draw conclusions about *other* states of affairs from the observation of some *given* state of affairs, using laws as premises. For example, we would not be entitled to make predictions. This would apply even if

35 So the question here is not (as Hume asked and answered in the negative) whether we can observe the necessity of a certain object falling down towards the earth. (It seems obvious that modal facts are not directly empirically accessible.) The question is rather whether what we observe, e.g., that a certain object falls downwards, is (conditionedly) necessary *as such*. Of course, one can hold – with Hume – that this question is meaningless because it is epistemically impossible to know any answer to it, namely, to know whether it is like that or not. But perhaps this view is wrong. Perhaps, while we cannot know any answer in the strict sense, we can still find that a particular *hypothetical* answer explains what we observe better than other hypothetical answers. But we would then have a reason to assume that what this answer expresses *is in fact the case* (e.g., that the object in question necessarily falls downwards *in fact*, and then in consequence, e.g., that there *in fact are* laws of nature). In this case the corresponding question would not be pointless.

the laws of nature were actually in force in our world. Of course, one may want to *epistemically* grant the possibility that we observe merely accidental regularities in our world, and not necessary regularities, because in our world the laws of nature are not valid at all (but rather, things just happen to behave in accordance with them). But if we exclude the possibility of concrete necessity from modal theory, then the difference between accidental and necessary concrete regularities objectively does not exist at all – so there is no epistemic undecidability at all, because there is nothing to decide.

Apodictically, Problematically, and Assertorically Conditioned Necessity

As already indicated, conditioned necessity may be formulated with regard to different modal contexts, e.g., with regard to nomological necessity, deontic necessity, juridical necessity, the rules of a game, etc. Likewise, the state of affairs which is set as a condition may have different characteristics. If, e.g., in the context of a game of chess, one says: "Peter must sacrifice his rook", then the condition which is implicitly set is not only the state of the match at that point in time, but also the fact that Peter wants to win the match. The pragmatics of problematically conditioned necessity allows for many distinctions and variations which I will not go into in this text.

However, there are three (and, it seems to me, only three) fundamental types that we can systematically distinguish within the field of conditioned material necessity, that is, with regard to the character of the condition's irrevocability. First, this condition may be irrevocable only in the sense that we *set* it as a condition, i.e., that we subjectively "insist" on this condition, as in the example of Peter's acceleration *given that* he is in free fall. In this case, the resulting necessity is *problematically* conditioned.

However, there are two – and, as far as I can see, only two – ways in which a set material condition of necessity may be irrevocable, in the sense of being objectively rather than subjectively set. We could say that in these cases, conditioned necessity is not only irrevocable, but *irreversible*. In one of these cases, this condition is *temporally* irreversible, because it lies in the past of that which is conditionedly necessary, and the past cannot be changed. This I shall call *factually* conditioned

necessity. Since, in this case, the condition is irreversible only *relative* to the conditioned, this conditioned necessity is still a *hypothetical* one, as is problematically conditioned necessity.

Then there is another type of condition that is not hypothetical but apodictical, in the sense that its irreversibility is not relative to *something else* – relative to later points in time, or relative to a subject's focus – but relative "to itself". This condition is the existence of something: relative to itself, its own existence is irreversible. Hence that which is necessary on the condition of the existence of something (and on this condition alone) is necessary *for* that thing *apodictically*: that thing cannot exist without the respective state of affairs being the case. This conditioned necessity can be called *dativic* necessity, and it is normally expressed by necessity *de re*. I will begin with this latter type of conditioned necessity.

2.2.1 *Apodictically Conditioned Necessity*

Colloquially speaking, apodictically conditioned necessity is necessity *for* a (grammatical) subject, since this necessity is *conditional* (*only*) *on the existence* of this subject. This means that this necessity is not given absolutely, but can only be eliminated by eliminating the subject. Therefore it is irreversible *for this subject*.

In ordinary language, we utter sentences like: "It is necessary for Peter to be at exactly one spatial location at a time", "It is possible for Peter to be married", "Peter must be spatially located", and "Peter could be married". This can be called "dativic" or "subject-relative" necessity, as opposed to nondativic or irrelative necessity (or time-relative necessity; see below). Dativic necessity *in our way of talking* usually corresponds to apodictically conditioned necessity *in the matter*. There is no possibility *for* Peter to be married if he does not exist (e.g., in a counterfactual possible world where he does not exist). There is no necessity *for* Peter to be spatially located if he does not exist. Dativic necessity is, so to speak, something between the (grammatical) subject and the predicate, with neither of these, it cannot be given. This is how I want to define dativic necessity. My contention is that this is what we normally understand when expressions such as those cited above are used in everyday pragmatic linguistic contexts.

According to an intuitive interpretation,[36] this distinction *formally* corresponds to the modal-logical distinction between necessity *de dicto* and *de re*. Sentences like "Peter is necessarily spatially located" and "Peter is possibly married" express the latter, in contrast to sentences like "Necessarily, every corporal individual is spatially located" and "It is possible that Peter is married". The former way of articulating necessity is called "*de re*" because it apparently speaks of a necessity in the thing itself, the latter "*de dicto*" because it apparently ascribes necessity, that is, necessary truth, to a statement. Viewed formally, the difference is the following. In the case of necessity *de re*, the modal operator comes after the subject about which something is said, i.e., the subject stands outside the scope of the modal operator, and something that is said about this subject falls into its scope; so that what is qualified modally is the respective *attribution to the subject* (typically the subject of a predicate), i.e., the "tie" between the subject and the predicate. In the case of necessity *de dicto*, on the other hand, the modal operator comes before the respective proposition as a whole, i.e., this whole proposition falls within its scope. Hence the modal operator qualifies *the truth of the statement*. Therefore we can also distinguish formally (and thus more precisely) between *predicative* and *propositional* necessity.

However, the question of whether formal necessity *de re* expresses dativic necessity in all its aspects depends on how proper names are treated in the respective formal language. As already noted, it is not the case that it is necessary *for* Peter (understood in the sense of dativic necessity) to be spatially located if he does not exist, since, if something does not exist, then no dativic modal states of affairs concerning that thing are given at all, including, e.g., the necessity *for* Peter to be identical to himself (which is different from the necessity *that* Peter is identical to himself). Depending on a formal language's grammar, it may be difficult or impossible to express this in that language. However, this need not concern us here, since we are investigating the pragmatics of ordinary modal language. Intuitively, most people would agree that Peter has no

36 Though not according to all (theoretically) possible interpretations; see below.

spatial location in a world where he does not exist. But he does have a spatial location in every world where he exists. The concept of apodictically conditioned necessity allows us to articulate the latter necessity while defending the nonnecessity of the *state of affairs* that Peter is spatially located: this state of affairs does not exist in those worlds where Peter does not exist. However, even if we do not want to defend this latter thesis, we still can differentiate those worlds where Peter exists while spatially located (i.e., where he is identical to something that exists while spatially located) from those where he does not exist (i.e., where he is not identical to something that exists), and hence does not exist while spatially located. That is, we still can distinguish dativic necessity *even if* we want to hold that "Peter is spatially located" is true at every possible world, regardless of his existence in that world.

As I have said, in some formal languages we may not be able to distinguish dativic necessity in cases where proper names are involved in our speech, depending on how these languages treat proper names. However, the distinction *as such* hinges (only) on the standard distinction in predicative logic between predication and existential quantification. (Nevertheless, the following differentiations do not depend on this specific formal analysis, but only on the assumption that, regarding the copula "is", we distinguish the sense of predication from that of existence, as is already the case with Aristotle.) A simple predicative sentence like "The 46th president of the United States is male" is to be analyzed in accordance with this distinction: "There exists the 46th president of the United States, to whom applies: *is male*." If the subject of the sentence does not exist, then according to this analysis the corresponding sentence is false in any case, regardless of the predicate. Hence "The current king of France is married" is false, because the following does not hold: "There currently exists the king of France, to whom applies: *is married*."

If we apply this distinction, e.g., to the fact that Peter is married, then we can (pragmatically, in ordinary language) distinguish two aspects: the aspect of Peter's existence and the aspect of his being married (even though formal languages typically do not allow us to express this distinction directly, since they do not allow for quantification over names). *Dativic* modality then implies that only the latter aspect is modally qualified. If we say that it is necessary for Peter to be spatially located, then we do not normally want to say that the state of affairs that Peter is spatially

located exists in every possible world (or, more formally: that Peter is identical to something that exists, while spatially located, in every possible world), because this state of affairs includes (the aspect of) Peter's existence. We normally want to express the thought that Peter is *necessarily tied to* his being spatially located. His existence, however, is unaffected by this necessity, it remains contingent.

According to this understanding, it is not the case that if it is true to say: "It is necessary for Peter (*de re*, in dativic interpretation) to be spatially located", then it is also true to say: "Necessarily (*de dicto*), Peter is spatially located." For the latter means that Peter is spatially located in all possible worlds, whereas the first sentence only states that he is spatially located in those worlds where he exists. Dativic necessity, therefore, must not simply be understood as the necessity (*de dicto*) of the state of affairs that is given when the corresponding subject exists while satisfying the corresponding predicate.[37]

If, instead of "Peter", we use an individual variable, as in: "Something is necessarily spatially located" (i.e., there exists something which is necessarily spatially located), then the case is formally clearer, but intuitively less obvious. By stating this, we say that there *actually* exists something for which it is necessary (i.e., it is necessary *for that very thing*) to be spatially located, i.e., that this very thing is spatially located in all possible worlds *where it exists*. This does not imply that such a thing exists in all possible worlds, i.e., that necessarily, there exists something which is spatially located. However, since many people intuitively also affirm the latter proposition, they may overlook the difference.

37 If one does not want to follow that analysis, then the following two options remain: a sentence like "The current king of France is married" makes sense but is undecided as to its truth value; or else it does not make sense (on its own). But this leads to problems when we talk about possible worlds: the statement "Peter is spatially located" is then true (as under the standard analysis) in all possible worlds in which he exists; but in all possible worlds in which he does not exist, its truth value is either undecided, or the statement does not make sense at all. But what does this mean for quantification over these worlds? Does it mean that, necessarily, Peter is spatially located? Or that this does not hold necessarily? Or that the truth value of "Necessarily Peter is spatially located" is indeterminate? Or that it is not a meaningful sentence? I prefer the standard analysis, under which our understanding of ordinary sentences, such as the above example, in modal contexts is clear from the outset.

Of course, it is *theoretically possible* to construct a theory where individuals which do not exist in a certain world still satisfy predicates in that world. According to such a theory, Peter would have a marital status and a spatial location in every world even when he does not exist in that world. However, my intuition tells me that such a theory would be highly artificial. Moreover, there would then be possible worlds that are exactly the same in every aspect *except* that, at a certain point in time, a nonexistent individual (in that world) occupies different spatial locations at the same point in time in those worlds.[38] Furthermore, we normally do not assume that nonexistent individuals satisfy (concrete) predicates when we talk about existence and nonexistence in the *temporal* sense: Most people would agree that the state of affairs that Socrates is asleep on New Year's Eve of 2024 does not exist, and nor does the state of affairs that he is awake at that time. And most people would agree that there is no concrete spatial location where Socrates is located at the present moment.[39] Hence most people will find it intuitive to say that the state of affairs that Socrates is asleep on New Year's Eve of 2024 does not exist in *any* possible world where Socrates does not exist at that time or where he does not exist at all – otherwise the actual world would have to be special in a weird sort of way.

Hence it seems intuitive to say that we can distinguish between the necessity of a *state of affairs* and the necessity of an individual's *satisfying a certain predicate*. And this distinction can be explained or made explicit by referring to the existence of the respective state of affairs in *all* possible worlds on the one hand, and on the other hand, to the state of affairs of the respective individual existing while satisfying the respective predicate *not* in all possible worlds but only *where that individual exists*. It seems evident to me that we intuitively make this distinction in our ordinary modal linguistic practice, i.e., that it is a typical modal pragmatic distinction. This observation is independent of the questions of whether and how formal languages can account for this distinction. However,

38 However, one may even accept this and still argue that in the sense of dativic necessity, there is no necessity *for* Peter to be at exactly one location at any point in time, if he does not exist.

39 Of course, some may say that he is "in heaven" or "in hell" or some such place; but those people imply that Socrates still exists, in some sense.

even if (contrary to what seems obvious to me) we would not make this distinction in *ordinary* linguistic practice, it is certainly a distinction that we can introduce and use, i.e., a distinction that we can clearly define on the basis of PWS. Since PWS allows us to distinguish all those elements that are necessary to establish this distinction, that is, to distinguish those possible worlds where, e.g., Peter exists (where there exists an individual that is identical to Peter) from those where he does not.

Expressed in PWS, the necessity *for Peter to be F* entails two things, namely (1) that Peter exists, i.e., that he is given in the actual world, and (2) that he is F in all possible worlds in which he exists (but not that he exists in all possible worlds while being F). We can avoid including the condition "in which he exists" in (2) by reformulating it as: there is no possible world in which Peter exists while not being F (more on this below). If the corresponding (true) statement contains an individual variable instead of "Peter", then: (1) there is (actually) some individual x, such that: (2) there is no possible world in which x exists while not being F. It is important that (2) is formulated as depending on (1). It is not enough to say: (1) there is some individual x, and (2) there is no possible world in which *some* x exists while not being F.

If the variable is bound not by an existential quantifier, but by a universal quantifier, then: (1) to all (actual) individuals (i.e., to all individuals in the actual world), the following applies: (2) there is no possible world in which any *of these individuals* exists while not being F. It may seem, at first sight, that in this case we can infer: $\forall x \Box Fx \rightarrow \Box \forall x Fx$. However, this is not the case – at least it is not theoretically-necessarily the case, depending on our metaphysical assumptions. To construct a counterexample to this inference, let us consider concrete individuals (only), i.e., let us take x from the set of concrete individuals. Then "$\forall x$" quantifies over all concrete individuals in the actual world – and not in all possible worlds, evidently. Let us further assume that all concrete individuals necessarily have the dimensionality they have. Peter could not be a 2-dimensional being, or a 121-dimensional being, in any possible world. However, all concrete individuals in the actual world have a certain dimensionality, or at least a certain range of dimensionalities – at least that seems intuitive.[40]

40 It is debatable whether concrete individuals in the actual world are all 3-dimensional (moving, but not extended through time), or if there are (also or exclusively)

Then, obviously, it would be true that, e.g., all (actual) concrete individuals are necessarily 3-dimensional (or 4-dimensional, or more-than-2-dimensional). But it would not be true that necessarily, all concrete individuals are 3-dimensional, because (intuitively) there are possible worlds with a number of dimensions different from that of the actual world, and where, accordingly, concrete individuals (necessarily) have a different dimensionality than actual individuals (necessarily) have. As I have said, one may reject the metaphysical intuitions on which this counterexample is based, but it seems sufficient that it is theoretically possible for us to construct such an example (i.e., we can construct such an example on the basis of a theory which is not obviously inconsistent) to falsify the logical validity of "$\forall x \Box Fx \rightarrow \Box \forall x Fx$".

Of course, in the consequent, we can then bracket the actual existence of x modally and say, e.g. that (1) *possibly*, there is some individual x *to which the following applies*: (2) there is no possible world in which x exists while not being F. For example, possibly there is something which is necessarily 24-dimensional.

The *possibility for* an individual x to be F means that (1) there is an individual x such that (2) there is a possible world in which x exists while being F. In this case, we are allowed to infer: $\exists x \Diamond Fx \rightarrow \Diamond \exists x Fx$, but we "lose" the actual existence of x in the consequent.[41] Hence the following is not valid: $\exists x \Diamond Fx \leftrightarrow \Diamond \exists x Fx$, at least not concerning dativic possibility as developed here. On the other hand, the following does not hold: $\forall x \Diamond Fx \rightarrow \Diamond \forall x Fx$. If, e.g., it is possible for all US-born US citizens to be the president of the US, it does not follow that it is possible that every one of them is the president of the US, i.e., that there is a possible world where all US-born US citizens are presidents of the US. The analogue applies to impossibility and nonnecessity.

4-dimensional concrete individuals, or if individuals are "secretly", e.g., 10-dimensional, because they have hidden dimensions (as discussed in string theory). In any case, the range of dimensionalities they may (actually) have is certainly limited. They are, e.g., necessarily "3-or 4-dimensional", or necessarily "more then 2-dimensional".

41 Hence the genuine contribution of apodictically conditioned possibility in this case is the connection of possibility to actual existence. It *genuinely* formulates this connection.

We see immediately that on this understanding, it is tautologically impossible *for* an individual not to exist. (As explained above, this is exactly why this type of necessity is called "apodictical".) In direct translation from ordinary language, the formal formulation of the possibility of nonexistence for x would be: "$\exists x \Diamond \neg \exists x$".[42] However, this expression is not considered well-formed in the usual formal logical systems because "x" is bound twice. It turns out that it is difficult to express the *impossibility for* x to be nonexistent in traditional formal languages – which, to me, does not seem to be a problem for dativic necessity, but rather for those languages. We could formulate it as (e.g.): "$\exists x \Diamond(\neg \exists y\ x = y)$", but, again, most formal logicians would not accept this formula as making sense. *If* we take it to make sense (which, in my view, is certainly possible), then it is important that "$\neg \exists y\ x = y$" *as a whole* falls within the scope of the possibility operator. "$\neg \exists y\ x = y$" is therefore a predicative expression (an expression with a blank space, namely: "$\neg \exists y\ \ldots = y$") – in contrast to "$\neg \exists x$" (for this alone is a *propositional* expression, and not a predicative one) – and the statement is: "There exists an x for which it is possible to be: $\neg \exists y\ \ldots = y$." In PWS this means, according to the above, that (1) there is an individual x, such that: (2) there is a possible world in which x exists while being: $\neg \exists y\ \ldots = y$. But in ordinary language "$\neg \exists y\ \ldots = y$" means: "… is identical to something that does not exist". However, something that exists cannot be identical to something that does not exist, in any possible world in which it exists. So "$\exists x \Diamond(\neg \exists y\ x = y)$" is clearly wrong, and this can only mean that it is impossible *for* any x not to exist. For possibility-for is apodictically conditioned possibility: the possibility under the condition of the existence of that for which the possibility is formulated.

However, the fact that the possibility of nonexistence does not exist *for* an individual should not be called "necessity" (or "impossibility") in the strict sense, but rather a "lack of alternatives" or "constraint": *For Peter* there is no alternative to his existence (he is constrained to exist), because *for him* it cannot be the case that he does not exist. We could also put it as: Peter *has* the possibility of not existing, but it is not possible *for*

42 If we want to allow for this formulation, we must distinguish between predicative necessity *de re* and propositional necessity *de re* ("$\exists x \Diamond \neg \exists x$" would be an example of the latter). This distinction would then be significant for various reasons, but I will not pursue this issue further here.

him not to exist – but, of course, ordinary language is ambiguous in these formulations. The reason why an individual's own existence should not be called "(apodictically conditionedly) necessary" (in the strict sense) is that its existence is *eo ipso* given in all possible worlds that are taken into consideration in evaluating this apodictically conditioned necessity (concerning this individual), since this existence is the very (material) criterion of this selection of worlds. The individual in question is already given in all these worlds, directly in virtue of the materially determined selection of these worlds – unlike, e.g., its spatial location in all possible worlds in which it exists. However, as already explained above, the reflection of the criteria of the material selection of a set of possible worlds *within* this set cannot express necessity in the strict sense (but only "Heidelberg necessity").[43]

43 From this follows an argument against the ontological proof of God, which, as far as I can see, has not yet been put forward and which, in my opinion, is definitive (if one accepts the formal analysis proposed here): This proof is based on an essential determination of God, i.e., on a determination that he has necessarily if he exists. The corresponding necessity is therefore, of course, dativic necessity (i.e., apodictically conditioned necessity). The *question* is whether $\exists x \Box Fx$ – namely (depending on the exact formulation of the proof) whether there exists a being which necessarily has all perfections (i.e., has the essential property of possessing all properties of perfection) – i.e., is defined as possessing all perfections. (It is not sufficient that it has all perfections *incidentally*, it must be defined by them.) The *question* is of course not whether $\Box \exists xFx$ (i.e., the propositional, unconditional necessity); but "$\Box \exists xFx$" is precisely the *answer* that one would like to arrive at (with *inferential* necessity), *starting* from the question. The concept of an *ens perfectissimum*, which is implicit in the question, is supposed to lead us to the answer that this being necessarily exists – this is the formal structure of the proof. Now we do not even have to discuss whether existence is a predicate, whether it is a perfection, etc. We can simply say that if dativic necessity is considered, then the existence of the subject of the predication in question is already assumed; it is *without alternative*. This type of necessity is *constrained* to the existence of the subject of predication. Necessity *de re* is conditional on this existence. It states (in PWS) that a certain predicate belongs to an (actually) existing (grammatical) subject in all possible worlds *in which it exists*. On the one hand, this implies, as I have said, a tautological lack of alternatives to its own existence *for* the subject. On the other hand, no predicate can modally attribute to the subject (i.e., *de re*) its existence (or anything else) *independently of this condition* (i.e., beyond those worlds in which this condition is met, i.e., in which the subject exists), because the modality (in this case, the necessity) of the attribution (i.e., *predicative* necessity) stands under the condition

Of course, the lack of alternatives *for* an individual to its own existence does not contradict the fact that this individual does not necessarily exist, i.e., that it could have not existed. But this cannot be expressed using "∃x ◊(¬∃y x = y)" (if we accept this formulation at all), because this formulation expresses apodictically conditioned possibility. To express the simple, unconditioned existence of what is not necessary, we can write, e.g.: "∃x, ◊¬∃y: x = y". In this case, the scope of the possibility operator only includes "¬∃y" ("◊¬∃y" is therefore a case of *propositional* and not *predicative* possibility, as already stated), and the statement is: There exists an x (in the actual world), and there is a possible world in which no y exists such that x and y are identical.[44] Of course, y must exist in the actual world if x exists in it. But that does not contradict the fact that y does not

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of this existence in the respective possible worlds. (It only quantifies over those worlds in which the subject exists.) To put it in simple terms: If it is necessary *for* God to be perfect (because that is how he is defined), then that may imply that it is necessary *for* him to exist (because existence is a perfection). But necessity-for is *already* under the condition of the existence of the subject in question, and it *always remains* under this condition. The (eventual) necessity *for* an individual to exist always remains under the condition that it exists (even abstracting from the question of whether existence is a predicate at all). In contrast, the *de dicto* question: "Is it necessarily the case that God is the bearer of the predicate of existence?" already implies the necessity of God's existence as the bearer of this predicate: "□∃xFx?" For it can only be the case that it is necessarily the case (i.e., in all possible worlds) that God satisfies the predicate F, if God *exists* in all possible worlds. The necessity of God's existence cannot be proven from this question, because the proof would then be circular. So the crucial step of the proof is: "∃x□Fx? → □∃xFx", and this step is definitely invalid, no matter what "F" says (and no matter how we interpret the question mark), on purely formal grounds. It is also invalid if we infer (accepting the unorthodox formal formulation that corresponds to ordinary linguistic usage): "x□(∃x)? → □∃x". On the other hand, it is necessary for (i.e., *de re*) *every* x that exists that it exists, or better: *for it*, there is no alternative to its existence. (Another argument which has already been put forward is: Even if we accept that ∃x□Fx → □∃xFx, when F = … exists, the truth of "□∃xFx" still depends on the truth of "∃x□Fx". That is, *if* God exists, then we can infer that he *necessarily* exists. However, this does not help us to decide the question of whether he exists. The mere predicative necessity "□Fx" (in ordinary language: "It necessarily exists"), with no quantifier, does not allow us to infer anything, since it is not a proposition.)

44 This is evident from the fact that we can exchange the first two expressions: There is a possible world in which no y exists, such that: there exists an x (in the actual world) and x and y are identical. In formal language, we could say: "◊¬∃y, ∃x: x = y"; or we could say: "◊¬∃y @∃x x = y", where "@" is the "actual-operator", that is a

exist in *another* possible world, in which of course x does not exist either. So x is identical to something that in *some* possible world does not exist, which just means that x does not necessarily exist.

It is important to note that if we understand "$\exists x \Diamond \neg \exists y\ x = y$" (without brackets) in the sense of "$\exists x, \Diamond \neg \exists y: x = y$", then the so-called Converse Barcan Formula (CBF): "$\exists \nu\ \Diamond A \rightarrow \Diamond \exists \nu\ A$" (which I implicitly used and affirmed above), is not applicable to it, i.e., "$\Diamond \neg \exists y\ x = y$" cannot be substituted for "*A*" in CBF, since in CBF "*A*" falls entirely within the scope of the modality operator, whereas in "$\exists x \Diamond \neg \exists y\ x = y$" (if understood in the sense of: "$\exists x, \Diamond \neg \exists y: x = y$", and not in the sense of: "$\exists x \Diamond (\neg \exists y\ x = y)$"), "$\neg \exists y\ x = y$" does not fall entirely within the scope of the modality operator. That is, if the formula "$\exists x \Diamond \neg \exists y\ x = y$" is understood to express the contingency of the existence of x, then CBF is not applicable to it.[45]

We see that according to what has been explained here, the predicative modal statement in dativic necessity is affirmed of the individual in question *as such*, and not of this individual under a specific description. (That is precisely the meaning of dativic necessity.) In other words, the condition of apodictically conditioned necessity is solely that of the *existence* of the modally predicated subject, not any description of it. If it is necessary for Mary's husband to be spatially located, then this is *simply* necessary *for him* (simply necessary on the condition of his existence), i.e., *for that individual* who happens to be Mary's husband, and not depending on his being her husband. The description "Mary's husband", in this context, being one of *dativic* necessity, only serves to single out the individual in question in the actual world. The apodictically conditioned necessity of being spatially located is then simply valid of it as existent, that is, *de re*. Accordingly, the following holds all the same: It is possible for Mary's husband to be unmarried. *For* the individual who is Mary's husband it is possible to be unmarried; being unmarried is compatible with their existence. Of course, this is not compatible with that individual being Mary's husband. But that is not what dativic necessity expresses. Of course, it is necessary that *if* x is Mary's husband, *then* x is married. (On the other

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rigidifying operator that always returns the semantic evaluation to the world of utterance.

45 This application would render the inference to: "$\Diamond \exists x \neg \exists y\ x = y$", which is obviously false.

hand, it is not necessarily (*de dicto*) true that (some individual who is) Mary's husband is married – that is, that in every possible world there is someone who is Mary's husband, to whom applies: *is married*. But that is a completely different statement.)

However, in everyday language, "Mary's husband is necessarily married", or even "It is necessary for Mary's husband to be married", is often understood in the sense of: "Since x is Mary's husband, x is necessarily married." This formulation with "since" or "insofar as" or "in view of the fact that" expresses *problematically* conditioned necessity, which I will discuss in the following paragraph. The "confusion" between apodictically and problematically conditioned necessity in normal language use is, on the one hand, due to the fact that language is notoriously vague, while the human mind is notoriously imprecise in its use. On the other hand, it is because we intuitively try to understand statements in such a way that (1) they make sense, and (2) the information given in the statement is relevant to its meaning (i.e., that even parts of the statement are not meaningless). For the statement that a certain person is not necessarily married, the information that this certain person is Mary's husband is irrelevant, because for every person it is true (in principle, in the unconditional sense) that they are not necessarily married. It would therefore be pointless to state this general fact that people can be unmarried *specifically* with regard to Mary's husband.[46] On the other hand, with regard to the statement that it cannot be the case that someone is unmarried, *in view of the fact that* he is Mary's husband, it is of course relevant that this individual is Mary's husband. Therefore, colloquially, we are inclined to understand "Mary's husband is necessarily married" in this latter sense

46 However, we can easily imagine a context of speech in which this way of speaking makes sense. Let us assume that Peter constantly exploits and mistreats his wife. Then, perhaps, a friend says to him: "Your wife is not necessarily married to you." He draws Peter's attention to the fact that his wife may possibly divorce him, i.e., that he cannot take it for granted that she will always be his wife. We immediately see that in this situation the sentence is easily and naturally understood in the "correct" way, namely in the sense of dativic necessity. So ordinary language does not require us to understand the necessity in "Your wife is not necessarily married to you" as a function of the subject's being married, that is, in the sense of: "Since your wife is your wife …" Quite obviously, ordinary language allows for the dativic understanding of this necessity, given the pragmatic circumstances.

(according to which it is true), although (according to what has been explained here) it should, strictly speaking, be understood in the former sense (according to which it is false), because it is only on the former understanding that this statement makes sense and its partial statements are meaningful. This may become clearer if we substitute "Peter" for "x" or "someone": "Peter, Mary's husband, is necessarily married" – that is obviously wrong. In this case, "Mary's husband" is of course superfluous, because the subject of the statement is already singled out by the proper name: the determination that Peter is (also) Maria's husband is not necessary to understand the modal statement. This is precisely why this additional determination cannot change the meaning of the modal statement.

Consequently, it is true to say, e.g.: "There is a bachelor of whom the following is true: he is possibly not a bachelor." Being a bachelor determines the selection of the individual in the *actual* world. But, of course, it does not determine the selection of *possible* worlds about which the modal statement is made. Otherwise, it would have to be true of an (actually existing) individual who is a bachelor that he is *necessarily* unmarried: that he could not have married and cannot marry in the future. Of course, the reverse is also true. Let us assume – just for the sake of the argument – that a person necessarily has the parents they actually have. Let us further assume that Anna is the mother of Mary's husband. Then we can say: "Mary's husband is necessarily the son of Anna", that is: "There exists (at time t) an individual x who is Mary's (only) husband, and x is necessarily the son of Anna." Of course, Mary could have married another person whose mother is not Anna. So it is not (problematically conditionedly necessarily) true to say: "*Since* x is Mary's husband, x is necessarily the son of Anna." But of the individual who is singled out in the actual world as being the (only) husband of Mary, it is true to say: "He is necessarily the son of Anna."

Therefore, *in the sense of apodictically conditioned necessity*, the following must hold: "The number of planets is necessarily greater than seven." That individual (assuming that numbers are individuals) which is singled out in the actual world by the description "... is the (only) number of planets (in our solar system)" is necessarily greater than seven, because this number is eight – or according to the old classification (i.e., counting Pluto as a planet), nine. However, Willard Van Orman Quine held that

the statement "The number of planets is necessarily greater than seven" should be understood in a sense in which it is false.[47] Under my interpretation this is a misunderstanding, which is not only due to the fact that language is notoriously vague, while the human mind is notoriously imprecise in its use of language, but also has to do with how names are treated in normal formal languages.

In such languages, quantifiers bind variables, but not names. Hence, e.g., "∃aFa" is not a well-formed formula in those languages, but "Fa" is. However, this leaves us with an ambiguity as to how the expression "□Fa" is to be understood: whether it is to be understood in the sense of necessity *de dicto*, e.g.: "Necessarily, Peter is spatially located"; or in the sense of necessity *de re*: "Peter is necessarily spatially located" (or: "It is necessary for Peter to be spatially located"). The first claim *would* be expressed by "□∃aFa" and the latter by "∃a□Fa", *if* these expressions were considered to be well-formed. "□Fa" looks more like necessity *de dicto*, since "Fa" falls entirely within the scope of "□". However, intuitively, we normally understand "□Fa" in the sense of dativic necessity: we merely understand that a is F in all possible worlds where a exists, not that a exists in all possible worlds (while being F). However, be that as it may, it is, in any case, crucial for *substitution* how we understand "□Fa".

To avoid the ambiguity of "□Fa", I will substitute it with either "□(∃xFx x = a)" (necessarily, a is identical to something which exists while being F) or "∃x□Fx x = a" (a is identical to something existing, for which it is necessary to be F), respectively.

According to the comprehension of *apodictically conditioned* necessity, the following inference is therefore valid:

(1) Eight is necessarily greater than seven (it is necessary for eight to be greater than seven): ∃x□Fx x = a, where: a = eight, and F … = … is greater than seven.
(2) Eight is the number of planets (in the actual world, in our solar system): ∃yGy y = a, where: G … = … is the number of planets (in the solar system).
∴

47 Quine, 1960, 41.

(3) For the number of planets (for that number which is the number of planets in the actual world, in our solar system), it is necessary to be greater than seven:[48] ∃yGy, ∃x□Fx: x = y (the actual number of planets is identical to something for which it is necessary to be greater than seven).[49]

Quine, contrary to this, understands the colloquial formulation of the conclusion (3) in such a way that it is not true, i.e., he understands it in the sense of (3*) "□(∃yGy ∃xFx x = y)". Accordingly, "□Fa" in (1*) must of course also be understood in the sense of necessity *de dicto*, and not in the sense of dativic necessity, otherwise this would not follow. This *de dicto* understanding of "□Fa" is in principle possible, since both "□(∃xFx x = a)" and "∃x□Fx x = a" are true regarding "eight" and "… is greater than seven". However, they are not the same with regard to substitution. Quine (implicitly) infers:

48 I admit that this formulation, at least without the addition in brackets, sounds wrong in ordinary language, just as "The number of planets necessarily is greater than seven" sounds wrong. This is, again, due to the fact that language is notoriously vague, while the human mind is notoriously imprecise in its use of it (and tries to understand utterances in such a way that its parts are meaningful for the whole: if someone simply wanted to say that nine is greater than seven, then it would be extremely odd to express this in such an elaborate way); and this is why it is important to analyze modal pragmatics. On the other hand, this shows that modal pragmatics is not about how utterances in ordinary language *must* be understood, but rather about what distinctions *can* be made in our understanding of such utterances: We evidently *can* distinguish between dativic necessity and nondativic necessity; as a consequence, we can show that necessity *de re* (in ordinary language) *can* be understood in the sense of dativic necessity. We can then establish that, for the sake of clarity, utterances of necessity *de re* should be understood in that sense, and stipulate that we will employ them in this sense in our discourse. Modal pragmatics does not claim any more than this, it is about possibilities of understanding, nothing else.

49 On this understanding (*de re*, in the sense of dativic necessity), the truth of "the number (which, in the actual world, is the number) of planets is necessarily greater than seven" (i.e., unlike the *de dicto* claim: "Necessarily, the number of planets is greater than seven", which is false) has nothing to do with the question of whether that number is singled out by a rigid designator or not. It only hinges on the fact that this number is singled out unequivocally *in the actual world*, e.g., by the description "the number of planets in the solar system".

(1*) $\Box(\exists x Fx\ x = a)$
(2) $\exists y Gy\ y = a$
$\therefore$
(3*) $\Box(\exists y Gy\ \exists x Fx\ x = y)$

However, it is quite obvious that this inference is wrong from the outset, since proper names in modal contexts *de dicto*, i.e., within the scope of a modal operator, cannot simply be replaced by descriptions that in the actual world pick out exactly the individual which is designated by the proper name: Indeed, this is so obvious that one cannot see why this should be a problem. For the following is valid: $a = b \rightarrow \Box(a = b)$; e.g., Hesperus and Phosphorus are the same in all possible worlds. But of course the following is not valid: $Fx \rightarrow \Box Fx$; e.g., if someone is married, it does not follow that it is necessary for him to be married; and if x is the number of planets, it does not follow that it is necessary – i.e., that it is the case in all possible worlds – that x is the number of planets. The following is therefore not valid: $\exists x Gx\ a = x \rightarrow \Box(\exists x Gx\ a = x)$ – if we can substitute a with x, which is G in the actual world, it obviously does not follow that we can substitute it with x which is G in all possible worlds. Consequently, even though (2) $\exists y Gy\ y = a$ in the actual world, "$\exists y Gy$" cannot be substituted for "a" in: "$\Box(\exists x Fx\ x = a)$", since "$\exists y Gy$" would thereby fall within the scope of the modality operator. However, if $\exists y Gy\ y = a$ in the actual world, this evidently does not guarantee that $\exists y Gy\ y = a$ in all possible worlds, as in: "$\Box(\exists y Gy\ \exists x Fx\ x = y)$". Contrary to this, "$\exists y Gy$" does not fall within the scope of the modality operator if we substitute it for "a" in "$\exists x \Box Fx\ x = a$". Hence the following is true: *For* that y which actually exists while being the number of planets, it is necessary to be greater than seven.

However, from the outset, Quine's planet example is not a good example of necessity *de re*, because the number eight (if numbers mundanely exist) exists in all possible worlds, and hence necessarily exists. But for entities that necessarily exist, *insofar as they necessarily exist*, dativic necessity is equivalent to the necessity *de dicto* of the state of affairs that exists when the corresponding entity is the bearer of the corresponding predicate: "It is necessary for eight to be greater than seven" is equivalent to "Necessarily, eight is greater than seven". But this equivalence does not hold simply, but only under the condition of the necessary existence of

the individual in question. For, as shown, "It is necessary for Peter to be spatially located" ($\exists x \Box Fx\ x = a$, where: a = Peter, and F … = … is spatially located) is not equivalent to: "Necessarily, Peter is spatially located" ($\Box \exists x Fx\ x = a$), because the state of affairs that Peter is spatially located is not given in those possible worlds in which he does not exist, according to the normal intuition. This means that in the case of entities that necessarily exist, predicative necessity *de re* is objectively no longer genuine; it does not differ from the corresponding necessity *de dicto* with regard to its objective significance. For the latter, however, the way in which the subject in question is singled out linguistically is relevant from the outset: Of course, it is not true (from the outset) to say: "Necessarily, the number of planets is greater than seven", because it is not true to say: "Necessarily, the number of planets is eight"; although "Eight is greater than seven" is equivalent to "The number of planets is greater than seven", *since* "Eight is the number of planets" is true – that is, in relation to the actual world. But statements that are equivalent with respect to the actual world need not be equivalent with respect to all possible worlds. The fact that "Eight is necessarily greater than seven" is not an example of genuine predicative necessity *de re* makes it easier (for Quine and all who agree with him) to understand this sentence in a sense in which "eight" cannot be substituted with "the number of planets": it makes it easier to understand "$\Box Fa$" in the sense of "$\Box \exists x Fx\ x = a$".

The equivalence "$\exists x \Box Fx\ x = a \leftrightarrow \Box(\exists x Fx\ x = a)$", *in the case that a, and hence also x necessarily exist*, is interesting in that "x" cannot be substituted in the same way in both of the "equivalent" formulas, because "$\exists x$" at first appears within the scope of the modal operator, and then not. We can interpret this as an indication that in logic, in some cases, the meaning of a sentence (which is of course different in the two expressions in the example) does matter, and not just its truth conditions. However, this difference in substitutability disappears when the condition of the necessary existence of x is integrated into the formula (as shown above), because then, also on the left side of the equivalence, "x" is within the scope of a modal operator: "$\Box \exists x \Box Fx\ x = a \leftrightarrow \Box(\exists x Fx\ x = a)$". This in turn shows that it makes a fundamental difference whether we say (*de dicto*): "Necessarily, eight is greater than seven" or (*de re*, understood as dativic necessity): "Eight is necessarily greater than seven", even though in this case (the case of a necessary existent) this objectively amounts to the

same, because eight exists in all possible worlds. But for the possibility of substituting for "eight", it does not amount to the same whether it is spoken of as existing in all possible worlds or only in the actual world.

To resume: Apodictically conditioned necessity is the necessity *for* something, which is given insofar as what is necessary cannot be suspended without suspending (the existence of) that thing. If Peter is necessarily spatially located, then that means that it is apodictically (i.e., *without alternative*) conditionedly necessary *for* Peter to be spatially located (the condition, namely his existence, is without alternative for him). For Peter, there is no alternative to being necessarily spatially located.

The semantics of the necessity (a) for Peter to be spatially located is modeled in PWS by selecting possible worlds according to a material criterion: In all possible worlds in which Peter exists, it is the case that Peter is spatially located. However, besides this, the actual condition of Peter's existence must still be maintained, because "In all possible worlds in which Peter exists, he is spatially located" is true even if he does not exist in the actual world – in which case necessity *de re* does not hold (see above). This also ensures that this necessity is, so to speak, "fulfilled" and not "empty": If it is necessarily *for* x to be spatially located, then it is also guaranteed that "In no world is it the case that x is not spatially located" is only true because x does not exist in any possible world. Because the necessity *for* something is conditional on the existence of that thing.

Analogously, we can formulate apodictically conditioned (b) possibility, (c) impossibility and (d) nonnecessity if we reformulate accordingly: (b) "There is a possible world in which it is the case that Peter is married "; (c) "In no possible world is it the case that Peter is ubiquitous"; and (d) "There is a possible world in which it is the case that Peter is not married".[50]

In (b) and (d) the criteriological selection of worlds (according to Peter's existence in them) is implicit: a possible world in which Peter exists is determined by a material criterion, namely Peter's existence in it. This is particularly relevant if the corresponding possibility or nonnecessity is counterfactual – otherwise the actual world is already of the type we are looking for. Or to put it another way, Peter of course knows that

50 As I have said, in addition, the apodictic condition, namely Peter's actual existence, must be stated.

he is possibly married if he is actually married (and knows this): he does not then have to "search" any more for possible worlds in which he exists; but if he is not married, then the possibility *for* him that he is married guarantees him that there is a world in which he exists while married.

In the case of *impossibility*, however, there seems to be no need for any selection of possible worlds. We can simply say: "There is no possible world in which it holds that Peter is ubiquitous."[51] To verify this, we do not have to verify if Peter exists in any world – this material criterion is irrelevant. On the other hand, the fact that Peter exists *at all* in some possible world is guaranteed by his actual existence (see above). However, there are possible worlds in which it does not hold that Peter is nonubiquitous, namely those in which he does not exist. "¬◊(∃xGx) x = a" is compatible with "◊¬(∃y¬Gy) y = a". For Peter this means that, on the one hand, his connection with "not being ubiquitous" is *noncontingent*:[52] the state of affairs that he is ubiquitous is *impossible*. On the other hand, his connection with "being nonubiquitous" is *contingent*: the state of affairs that he is nonubiquitous (i.e., that he exists while being nonubiquitous), is *not necessary*. Therefore, to unequivocally formulate the impossibility for a to be ubiquitous, the expression "¬◊(∃xGx) x = a" is not sufficient; and we must add that "◊¬(∃y¬Gy) y = a" only applies to the selection of those possible worlds in which a does not exist (i.e., in which the apodictic condition is not given – the selection turns from a positive into a negative one – but according to the same criterion).[53] We can also write

51 In the following: G = … is ubiquitous, and: a = Peter.

52 I use the term "noncontingent" in the sense of "either necessary or impossible", and the term "contingent" in the sense of "possible yet nonnecessary", i.e., not in the sense of "nonnecessary" only.

53 Of course, we could say, if "∃x, ¬◊∃yGy: x = y" is true, then it follows that "∃x, ◊¬∃y¬Gy: x = y" concerns only those possible worlds where x does not exist. However, this is exactly what makes "¬◊∃xGx x = a" and "◊¬∃y¬Gy y = a" compatible: the latter is restricted to worlds where a does not exist. Hence we can say that "¬◊∃xGx x = a & ◊¬∃y¬Gy y = a" expresses a materially conditioned selection of possible worlds: there are worlds where a does not exist. It *implicitly* formulates a selection by way of *de dicto* modality that is *explicitly* formulated using *de re* modality (even though the selection is "inverted": in the first case, all those worlds are selected where a does not exist; in the second case, all those where it does exist; but, as already stated, the material criterion – the question of the existence of a – is the same). Perhaps this becomes clearer by contrast: As explained above, in the

it the other way around: "¬◊(∃xGx) x = a & ◊¬(∃y¬Gy) y = a" expresses the contingency of a – and at the same time it expresses the apodictical conditionality of the non-G-ness of a. But the contingency of a indicates that there are worlds in which a does not exist – it implies that there are worlds to which the following material criterion applies: a does not exist in them. That is to say, in the end, in the case of apodictically conditioned necessity, we cannot avoid selecting possible worlds according to a *material criterion*, namely the criterion of the existence or nonexistence of the individual in question in them – at least if we want to formulate dativic impossibility *unequivocally*.

The meaning of "apodictic" in the above cases is the following, respectively: (b) For conditioned possibility, apodicticity means that the apodictically conditioned possibility is compatible with Peter's existence in the respective possible world. In precisely this sense, the possibility that Peter is not spatially located is not given apodictically, because it is incompatible with his existence in the respective world (in which this possibility is realized). It is indeed a possibility that concerns him (there are possible worlds where he is not spatially located), but not one that exists *for* him (in the dativic sense, as developed here). (c) For impossibility, apodicticity means that there are entirely possible worlds in which, e.g., it does not hold that Peter is nonubiquitous – namely those possible worlds in which he does not exist. But if he exists, then the state of affairs that he is nonubiquitous cannot *not* exist. Precisely for this reason, the apodictically conditioned impossibility that Peter is ubiquitous is stronger than the unconditional impossibility that Peter is ubiquitous. The latter is compatible with the possibility that the following does not hold: Peter is not ubiquitous, because there are possible worlds in which Peter does not exist. For the former, on the other hand, these worlds do not come into consideration. For Peter, it is impossible that the following

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case of individuals which necessarily exist, necessity *de re* is no longer genuine, because in this case there is no more (limiting) selection of possible worlds, since those individuals exist in all possible worlds. Hence, in this case, necessity *de re* is equivalent to the respective necessity *de dicto*. But this means that in this case "¬◊∃xGx x = a & ◊¬∃y¬Gy y = a" is not true. Suppose, e.g., that a is the number eight, and that G is the property of being a hedgehog. Since numbers exist in every possible world, there is no world where it is not the case that there exists something identical to eight which is not a hedgehog.

does not hold: he is not ubiquitous. (d) The meaning of apodicticity in the case of nonnecessity becomes clear as follows: The state of affairs that Peter is spatially located is, as I have said, not necessary because it is not given in all possible worlds – namely not in those in which Peter does not exist. *Apodictically conditioned nonnecessity* is so called because it is different from *this* nonnecessity: the state of affairs that Peter is married can not only not exist if Peter does not exist, but also if he does exist: Peter can exist and not be married. The possibility of his not being married is *compatible* with his existence (in the respective world), while the possibility that it is not the case that Peter is spatially located exists but is *incompatible* with his existence. In this case, "apodictic" means, so to speak, that we cannot eliminate the "possibility that not" from the world (or from the worlds) by making Peter's existence a prerequisite, as it works when we are dealing with the state of affairs that Peter is spatially located. This "possibility-that-not" exists *for Peter* – in contrast to the possibility that it is not true that he is spatially located.

We can see that if the modality in question is to be based on quantification over those possible worlds in which the subject in question exists, then predicative modality must be transformed in such a way that the corresponding proposition is preceded only by the necessity or possibility operator *without* negation, while the required negation in the case of possibly-not and necessarily-not remains with the predication. Intuitively, this is explained as follows: There is not actually any *impossibility for something* – e.g., for Peter, the impossibility of being ubiquitous – nor is there any *nonnecessity for something* – e.g., for Peter, the nonnecessity of being married – but what is given is positively the *necessity* of the negated predicate – namely, for Peter to be nonubiquitous – or the *possibility* of the negated predicate – namely, for Peter to be unmarried. The deeper explanation is that impossibility and nonnecessity level the difference between those possible worlds in which Peter exists and those in which he does not exist, i.e., they make the materially conditioned selection of the possible worlds on which quantification is based meaningless: Peter is, in no possible world in which he exists, ubiquitous; just as he is not ubiquitous in any possible world in which he does not exist (and likewise for nonnecessity). However, this means that predicative modality is no longer *genuinely* articulated *as such*.

As I have said, (simple) apodictically conditioned necessity stands under two conditions: First, under the condition of the actual existence of the individual in question, i.e., in the actual world; and secondly, under the materially conditioned selection of the possible worlds that are to be taken into consideration for the determination of necessity, namely under the criterion of the existence of the individual in question in them.[54] However, since for an individual, its existence is without alternative, for it the first of these conditions is fulfilled through it: the condition of the actual existence of the individual in question. Therefore we can determine dative necessity, that is, *necessity for* the individual in question, *simply* under the materially conditioned selection of the possible worlds that come into consideration for the determination of necessity according to the criterion of the existence of this individual in them. However, this only applies to concrete, actually existing individuals as such, i.e., *de re* – not to an individual under a certain description "Fx", i.e., *de dicto* (see above), because then this description (including all its implications) would automatically apply "necessarily" to x.[55] Precisely for this reason, apodictically conditioned necessity, as modeled here, is apt for genuinely representing *necessity for*, or *possibility for*, an actually existing individual *as such*, i.e., independently of its characterization, that is, as a necessity or possibility which actually exists for this individual (and not: which exists for it *depending on* a certain description): "There is the necessity for Peter to be spatially located" (in all worlds that are possible for him, he is spatially located), "There is a possibility for Peter to be married" (there is a world which is possible for him, i.e., in which he exists, in which he is married), etc.

Now, since apodictically conditioned necessity is apt for *genuinely* modeling the necessary (or possible) attribution of properties to a

54 Or alternatively, as just shown, the negative materially conditioned selection (i.e., the *exclusion*) according to the criterion of the nonexistence of the individual in question within them.

55 We may very well pick out the individual about which we want to state dativic necessity via a description, as in the examples above. However, the dativic necessity then concerns that individual *as such*, and not the description which (only) served to single it out. This is the very meaning of dativic necessity: it is only conditional on the existence of the individual in question, nothing else.

subject, it is *ontologically* useful if we want to defend an ontology according to which there are essential properties – e.g., the property of being spatially located. So the conception of apodictically conditioned necessity may be helpful in ontology, if we want to speak of essential properties (or relations) in it.[56] It should be remembered that this is not only necessary if we want to defend, e.g., Aristotelian essences in ontology, but also if we want to consider it as ontologically given that every material being is (necessarily) spatially located. If one does not want to speak of essential properties, then talk of dativic necessity is ontologically meaningless from the outset.[57]

From a modal-theoretic point of view, however, the following still holds: If we start from PWS and assume that one possible world is marked out as the actual one (even if only indexically, as the world in which *we* find ourselves), then everything is given that we need for modeling apodictically conditioned necessity.[58] There is, therefore, at least

56 Specifically, dativic necessity may be useful if we want to design a modal ontology where possible worlds (or something similar) have some sort of ontological status, and we want to avoid the consequence that, e.g., if Peter is essentially spatially located, then he is spatially located in every possible world, even in those possible worlds where he does not exist. This consequence may not matter much for possible world *semantics*, but it may be considered to be awkward for possible worlds *ontology*, since it implies an unnecessary proliferation of possible worlds.

57 Since the question of rigid designators does not matter for apodictically conditioned necessity, we can define the essential properties of individuals without having to resort to such designators. We need only resort to the identity of an individual with itself in all possible worlds, i.e., to the "transworld identity" of individuals. Of course, it is debatable whether identity and, more specifically, transworld identity, is an essential property. If it was, then the explanation of essential properties with recourse to apodictically conditioned necessity would be circular (which does not necessarily imply that it would be meaningless). However, transworld identity does at least seem to be something more fundamental than just an essential property, since its instantiation does not constitute (simply) an intramundane state of affairs – as all instantiations of all (other) essential properties do – but an intermundane state of affairs. However, if transworld identity is not a property (since "properties" are defined as intramundanely instantiated), but something more fundamental, then "essential properties" may be defined noncircularly with recourse to such identity.

58 Of course, we still need the concepts of "intermundane identity", and of a "selection of possible worlds under a material criterion", but it seems to me that we can hardly reject these, once we have accepted PWS together with actuality.

prima facie, no good reason to exclude apodictically conditioned necessity from ontology *from the outset*, if we allow for possible worlds (in some form), together with actuality. It thus seems (ontologically) nonsensical to want to ontologically *prohibit*, e.g., the property of being spatially located from being an essential property of (certain) individuals: a property that apodictically conditionedly necessarily exists for them.[59] It thus seems pointless to fundamentally oppose any form of essentialism, as is in vogue these days.

Notice that apodictically conditioned necessity, when ontologically admitted, modally links certain predicates to individuals *directly* or (in the sense presented above) *unfoundedly*, and not mediated by or founded on – that is, due to or under the condition of – another predicate. This is completely unavoidable, otherwise we would end up in an infinite regress when we attribute apodictically conditioned necessity. For if that on the basis of which we wanted to attribute a predicate *with necessity* to something did not in turn necessarily apply to that thing, then the inference to the necessary attribution of the first predicate would not be valid. This does not mean that *all* predicates that necessarily belong to an individual must necessarily belong to that individual *unfoundedly*. There may well be predicates that are necessarily attributed to it on the basis of other predicates. But ultimately, there must be an unfounded necessary attribution of predicates, if there is to be apodictically conditioned necessity.

If something has an unfoundedly essential property F, then this means that, figuratively speaking, it did not enter the actual world – or any possible world, or the space of possible worlds – without being F, and *then* acquire F in all those possible worlds into which it had entered (i.e., where it exists). Rather, this thing only entered logical space *while already being F*. An unfoundedly essential property is not acquired by or attributed to something *because of* a determination that it (only) has intramundanely, but rather belongs to it directly as what it is – i.e., *as what* it enters the space of possible worlds ("enters", of course, being understood here in an atemporal, "(onto)logical" sense). The intuition behind this is that something can enter logical space only *as something* (at all), i.e., it

59 Of course, one can deny on nonontological grounds that there is a property (or relation) of being identical with oneself. See, e.g., Wittgenstein, *Tractatus* 5.5301.

cannot enter logical space without any properties at all.[60] However, those properties with which it enters logical space are, *eo ipso*, unfoundedly essential properties. We can formulate this as: "As something enters logical space, it is something (specific)" – which is of course circular, but it expresses the fact that no relationship of foundation is given here – i.e., it says that unfounded necessity is being articulated.

One can of course be skeptical about unfounded apodictically conditioned necessity. However, we already have seen that, e.g., the laws of nature, if they are to be valid intramundanely with necessity, must be necessarily valid *unfoundedly*. The unfoundedness of necessity is therefore not a special problem for apodictically conditioned necessity or for essential properties. Of course, one can also reject the laws of nature, or at least their necessity. But that does not seem particularly intuitive, since the laws of nature are a typical case of our everyday talk of necessity.

2.2.2 *Problematically Conditioned Necessity*

Problematically conditioned modality is not under the condition of the existence of a subject, but rather under that of a particular predicative or relational state of affairs. This condition must be *sufficient* for problematically conditioned necessity, and *necessary* for problematically conditioned possibility. Colloquially, this necessity is formulated (as already indicated) as follows: "*Since* Peter is a bachelor, he is necessarily unmarried", or: "*Given that* Peter is a bachelor, …", or: "*In view of the fact that* Peter is a bachelor,. …" In most cases, however, the condition is not explicitly formulated in colloquial talk, but is instead implicitly assumed: "You can still catch the train (in the current circumstances)", "Peter must be in a lot of pain (since he has a certain illness that is necessarily associated with great pain)".

Problematically conditioned necessity is explained as follows: It is (unconditionally) necessary, i.e., in all possible worlds, that *if* the following is true regarding x: x is a bachelor, *then* the following is true: x is unmarried. If Peter is *actually* a bachelor, then of course it does not hold that he is *unconditionally* necessarily unmarried, because it is not necessary that he is a bachelor. But it is still *conditionedly* necessary that

60 This amounts to the intuition: "No entity without identity!", if we understand identity in the Leibnizian sense, i.e., as articulated by properties.

he is unmarried given that he is a bachelor. However, this is not *apodictically* conditionedly necessary, because being a bachelor is not necessarily linked to Peter's existence. In this case, the conditioned necessity is only a problematic one. It is nevertheless a real material necessity – and not just an inferential one – insofar as it distinguishes Peter's status of unmarriedness given his bachelorhood from that of his unmarriedness given that he is a resident of Krämergasse 2 in Heidelberg, in our example above.

In the case of problematically conditioned necessity, we allow ourselves to conclude: $\Box(A \rightarrow B) \rightarrow (A \rightarrow \Box B)$. This conclusion is of course not unconditionally valid (instead, K: "$\Box(A \rightarrow B) \rightarrow (\Box A \rightarrow \Box B)$", is unconditionally valid). But it is valid under the posited or set condition *that A* – i.e., it is valid with respect to all possible worlds (at all those points in time) at which A.[61] The *set condition* is therefore one (1) whose state of affairs is actually given, and which (2) is taken as the criterion of selection for those worlds over which the modal operator in "$\Box B$" quantifies. This modal speech practice makes sense, because otherwise (in the respective cases) we could not speak of the necessity of B, but could only say "$\Box(A \rightarrow B)$", and consequently could only speak of a general modal nexus. We could not modally qualify the state of affairs that results from the actual givenness of A under the presupposition that $\Box(A \rightarrow B)$. However, in everyday life we are often interested in the modal status of such concrete states of affairs. We are often interested in exactly that modal difference that exists, e.g., between the fact that Paul McCartney was a British citizen (more precisely: was born a British citizen) *since* he was a member of the Beatles (which is *inferentially* necessary, because all members of the Beatles were in fact – but not necessarily – British citizens, but which is not *problematically conditionedly* necessary), and the fact that he was born a British citizen *since* his parents were British citizens (which is *problematically conditionedly* necessary[62] – his parents were not necessarily British, but necessarily, under British law, the children of

61 We could express this by writing, e.g.: "$\Box(A \rightarrow B) \rightarrow (\underline{A} \rightarrow \Box_A B)$", where "$\underline{A}$" indicates that "A" is set as a condition, and "$\Box_A$" expresses problematically conditioned necessity relative to A. However, there will be no need to formally express problematically conditioned necessity in this text.

62 In this case, the necessity in question is of course not one of natural law, but rather one arising from the relevant national legislation. But that is not important for the difference that I would like to illustrate here.

British parents receive British citizenship at birth). And it is often important for us to emphasize that there is a necessity in the latter sense: Peter *must* be unmarried, since he is a Jesuit, but not, for example, since he lives at Krämergasse 2, Heidelberg. This difference is of course not an unconditional one. But it can be clearly and distinctly formulated, e.g., on the basis of PWS. There is therefore no reason why this speech practice should not be permitted, even in scientific discourse, as long as it is clearly and distinctly differentiated from other speech practices, e.g., from talk of unconditional material necessity.

In practice, problematically conditioned necessity plays a vital role in explanations: If all the residents of Krämergasse 2, Heidelberg, are (actually) unmarried and Peter is (actually) such a resident, then it follows that he is (actually) unmarried, i.e., this reasoning is valid and hence useful for ascertaining that Peter is unmarried. ("Peter must be unmarried", Sherlock Holmes concludes – in the sense of inferential necessity.) But that does not *explain* anything. If, however, Peter is a Jesuit, then this *explains why* he is unmarried: because necessarily, all Jesuits are unmarried. Peter is not only unmarried because, coincidentally, he has not found a partner, but because he has decided to enter the Society of Jesus and remain celibate. By contrast, mere inferential necessity is sufficient for *proof*, but on its own is not sufficient for *explanation*.

The difference between *apodictically* conditioned necessity and *problematically* conditioned necessity, as I have said, is that for Peter, there is no alternative to his being spatially located. On the other hand, if he is a bachelor, then he *does* have the hypothetical alternative of being married. It is true that he cannot be married *under this condition*, but since this condition can be eliminated without eliminating Peter himself, it is *simply* possible that he is married – it is impossible only under the aforementioned condition. On the other hand, it is simply impossible for him not to be spatially located.

The hypotheticality of the necessity of being unmarried is particularly interesting for Peter because his contingent predicates can (in principle) change *over time*. This can also be seen in formulations such as "Peter cannot be a bachelor and be married *at the same time*" and "Peter cannot be married *as long as* he is a bachelor". Peter need not therefore be very concerned that he cannot be married *since* he is a bachelor, because he can easily remove this condition, that is, by getting married – the

necessity only exists as long as the condition is *present*: as long as it is copresent with the conditioned. However, in reality, the cancellation of this condition may not always be so easy, hence problematically conditioned necessity is not always trivial and insignificant. For example, it is nomologically necessary that bodies in free fall above the earth accelerate at 9.81 m/s^2 (if we abstract from air resistance). Hence, if it is true that Peter is a physical being and is in free fall above the earth, then it is problematically conditionedly necessary that he accelerates towards the center of the earth at 9.81 m/s^2. Of course, it is not unconditionally necessary that he accelerates (he does not accelerate at the respective time in all possible worlds, not even in all the possible worlds in which he exists), but Peter may actually have difficulties in removing the condition that he is in free fall. It is therefore nontrivial for Peter to know this conditioned necessity, because he thereby knows that it is necessary for him to remove this condition in order not to accelerate any further. By way of comparison, Peter does not have to move out of his apartment at Krämergasse 2 in Heidelberg in order to stop being a bachelor.

Problematically conditioned modality enables us to concretely bring to bear sufficient or necessary conditions. Problematically conditioned *necessity* expresses the necessary existence of a state of affairs under the copresent givenness of a *sufficient* condition. Accordingly, problematically conditioned *impossibility* expresses the necessary nonexistence of a state of affairs in the copresent absence of a *necessary* condition: Since Peter is unmarried, it is impossible that he is Mary's husband. Problematically conditioned *possibility* expresses the possibility of a state of affairs under the copresent givenness of a *necessary* condition, i.e., a state of affairs in the absence of which the former state of affairs would actually be impossible: Since Peter knows French, it is possible that he reads Proust in the French original. If he could not read French, it would not be possible for him to read Proust in the French original, that is, not during the existence of the state of affairs that he does not know French. Of course, there is the *absolute* possibility that Peter can read Proust in the original, because there are possible worlds in which he can speak French (including, of course, those in which he also actually reads Proust in the original). And there may also exist the respective *practical* possibility, because he can learn French, so it is possible that he will know French in the future (but not in the temporal presence of the validity of

the above statement). Finally, problematically conditioned *nonnecessity* expresses the nonnecessity of a state of affairs under the copresent nongivenness of a *sufficient* condition, i.e., a state of affairs under which the former state of affairs would be necessary: Since Peter is not yet 70, he does not have to renew his U.K. driver's license yet. In everyday language we often speak of problematically conditioned necessity without specifically mentioning the condition: "It is possible for Peter to read Proust in the French original", "Peter does not have to renew his license yet", etc. Conversely, this means that this common speech practice has its basis in problematically conditioned necessity, which therefore appears to be modal-pragmatically relevant.

In ordinary linguistic practice, the identification of problematically conditioned modality, and specifically of problematically conditioned possibility, is often further complicated by things that we do casually in that practice, often without noticing. And we are often imprecise; we imply that we speak about relevant information only; we leave out premises and jump inferential steps. Hence, e.g., we would normally say that Peter can make a fruit tart, since he has strawberries, blueberries, and brambles. But, of course, he also could make a fruit tart if he had bananas, kiwis, and mangoes instead. Hence the former is not a necessary condition, and neither is the latter. The necessary condition to which we implicitly refer in this case is that Peter has *some* fruit. If he has strawberries, blueberries, and brambles, then he has some fruit. Hence, in this case, the necessary condition for making a fruit tart is met, even though having strawberries, blueberries, and brambles, in itself, is not a necessary condition for this. On the other hand, we would not normally say that Peter can make a fruit tart since he has flour and fat, since these are necessary for making tarts in general, not only fruit tarts. Hence the specific information that the tart he can make is a fruit tart is normally irrelevant in this context, and that sentence sounds odd. However, if, e.g., Peter has a fruit farm on a lonely island in the Pacific where he cannot get flour and fat, and then a visitor brings these to the island, then that sentence makes sense. For the same reason of "irrelevant information", we would not say that Peter can make a fruit tart since two plus three equals five, even though it is true that he does not make any fruit tart in any world where two plus three does not equal five. Of course, we could somehow try to build this condition into our definition of problematically

conditioned possibility, e.g.: "The respective necessary condition must not be given in all possible worlds." But since the pragmatic criterion of informational relevance is a general one in ordinary linguistic practice, it seems unnecessary and even inappropriate to formulate it specifically for problematically conditioned possibility.

As I have said, the fact that problematically conditioned possibility is only *problematically* given means that the condition in question is only a necessary one. Hence, even if the selected condition is objectively given, the respective possibility may at the same time not be given with regard to a different (objectively given, subjectively selected) context of consideration: Since Peter is currently on vacation at the home of a friend (on a lonely island without internet access) who only has Proust's works in English translation and not in the French original, it is currently impossible for Peter to read Proust in the original. This latter statement does not falsify the problematically conditioned possibility that Peter can read Proust in the original *since* he knows French, even if it does tell us that *in fact* he cannot do so at the moment.[63]

Problematically conditioned possibility depends on a condition which may very well be objectively given, but which is set as a condition only problematically. This may seem counterintuitive at first, but if problematically conditioned necessity were to be formulated on the basis of *all* the conditions that actually prevail at a given point in time, then no counterfactual problematically conditioned possibilities could exist at all, because the nongiven cannot coexist with the given. The conditional possibility that formulates counterfactual possibilities under the condition of the totality of what is presently factually given (albeit in another sense) is not problematic but is a *factually* conditioned possibility (see below).

Because of all this, problematically conditioned necessity and possibility are irrelevant to ontology. What is sometimes said about modality *de dicto* in fact applies to problematically conditioned modality: it is determined solely by how we speak of something. If we speak of Peter *in*

63 In this case, we would say in ordinary language, e.g.: "Well, then it is currently not in fact possible for Peter to read Proust in the French original, but since he knows French, it *would* be possible for him – unlike his friend whom he is visiting, for whom it would not be possible since he does not know French."

view of his being a bachelor, then he is necessarily unmarried. If, in contrast, we speak of him in our above example *in view of* his being a resident of Krämergasse 2 in Heidelberg, then he is not necessarily unmarried. If we talk about him *in view of* his knowing French, then it is possible that he reads Proust in the French original. If we speak of him in *view of* his being on a lonely island where Proust's works only exist in translation, then it is *impossible* that he reads Proust in the original. The nexuses under the premise of which problematically conditioned necessity is inferred may well be real, in the sense that they do not depend solely on how a certain term – e.g., "bachelor" – has been defined in language. Compare our example of the necessity of acceleration in free fall (the law of mass attraction is not a linguistic convention). But it is not *objectively* necessary that Peter is unmarried (since he is a bachelor), just as it is not *objectively* necessary that he accelerates at 9.81 m/s^2. (Rather, this is only necessary given that he is in free fall above the ground, which, however, is not necessary, at least not unconditionally, since it is not the case in all possible worlds.) This can also be illustrated by the fact that Peter's necessary acceleration, *since* he is in free fall, does not rule out the possibility that Peter will open his parachute in the next moment and cease to accelerate.[64] If, on the other hand, we want to express the *future* necessity of Peter's acceleration in free fall – because he does not have a parachute – then *problematically* conditioned necessity is not sufficient, and we will have to formulate *factually* conditioned necessity; see below.

One might want to object: "If Peter is now in free fall, then it is *objectively* impossible that he is not in free fall, and therefore it is also objectively necessary that he accelerates." But *this* impossibility does not correspond to any material modal state of affairs at all, but (from a material point of view) only to the factuality of Peter's free fall. That statement is necessary only in the sense of inferential necessity: if something is the case, then it follows logically (and therefore as a matter of inferential necessity) that it is not not the case, and if something is the case at a certain point in time (e.g., now), then it cannot not be the case at that point in time (in the sense of inferential necessity). But this does not result in any material necessity, not even conditioned necessity. Hence the fact that what

64 And, of course, it does not rule out the possibility that Peter is not in free fall in another possible world at the same time.

is the case (at a certain point in time) cannot not be the case (at that point in time) does not constitute any form of objective irreversibility or unalterability, as was the case with apodictically conditioned necessity (Peter's existence is irreversible and unalterable *for him*), and as will be the case with factually conditioned necessity. In the case of problematically conditioned necessity, the condition is unalterable only inasmuch as it has been set as a condition. However, since this setting is subjective, it does not provide any objective reference point for conditioned necessity, as was the case with apodictically conditioned necessity, and will be the case with factually conditioned necessity. Our setting Peter's free fall as a condition remains a purely subjective setting, even if it is objectively true that he is in free fall, and also objectively true that bodies that are in free fall above the earth accelerate at 9.81 m/s^2.

We can say that the selection of possible worlds that is taken as a basis in the case of problematically conditioned necessity is not *objectively indicated*, or that it is not without alternatives under an *ontic* criterion – as was true for apodictically conditioned necessity in the previous section (because it is not possible *for* an entity not to exist), and as will be true for factually conditioned necessity in the following section (albeit in a different sense than in the case of apodictically conditioned necessity). Therefore problematically conditioned necessity is irrelevant to ontology from the outset (but of course, not vice versa), while the other two types of conditional necessity *may* have ontological relevance, depending on how one devises one's ontology.[65]

65 This shows that the *ontological* meaning of the laws of nature cannot stem from or be articulated by problematically conditioned necessity. As will be seen, this can only be articulated by *factually* conditioned necessity – that is, as will be seen, in the context of time. This may seem confusing at first, especially since time plays no role at all in many scientific formulations of natural laws. But notice: It is certainly *simply* true that when a body is in free fall above the earth (in a vacuum), it accelerates at 9.81 m/s^2, i.e., without regard to time. But this only expresses the axiomatic necessity of the law of gravitational acceleration (with reference to the earth) within the world – i.e., this axiom as necessitated. However, the application of this general connection to a specific individual does not result in any objectively given modal state of affairs (e.g., it does not result that Peter *necessarily* accelerates at 9.81 m/s^2; he only *actually* accelerates at 9.81 m/s^2), but only results in *problematically* conditioned necessity (which is not an objectively given necessity), namely, *in view of* … (e.g., the fact that Peter is in free fall). The laws of nature – including

On the other hand, as I have already said, problematically conditioned necessity is often *epistemically* extremely illuminating: The fact that Peter is in free fall *explains* why he accelerates at 9.81 m/s^2. So this necessity is not subjective in terms of its content. Only the aspect, or the selection of the condition under which the necessity in question is given (and may then be objectively given), is subjective: Free fall explains Peter's acceleration from a physical point of view. This means that the point of view or the context of consideration decides about necessity (in the case of problematic conditioned necessity), but not because this necessity is not objectively given, but because it decides about the condition (that is, the problematic condition) under which the necessity is set. For example, if I see Peter falling past my window and I wonder why he is doing that, then it is typically not the physical explanation (that he accelerates at 9.81 m/s^2 due to Earth's gravity since he is in free fall) that I am looking for. I am interested in problematically conditioned necessity, but not considering (purely) physical conditions. However, this means that, from *that point of view* in which I would like to have Peter's fall explained to me, as far as I know, it is still nonnecessary, namely unexplained. It *will be* explained, for example, if someone tells me that Peter is a base jumper and his fall

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the "nontemporal" ones – therefore require time in order to *ontologically* articulate their (intramundane) necessity with regard to concrete individuals – in the broadest sense, including waves, fields, etc. – namely as a *factually* conditioned necessity; see below. However, we can conversely affirm that the laws of nature can *in fact* be articulated ontologically with regard to concrete individuals exactly via factually conditioned necessity. Necessity under natural law can *in fact* be attributed to concrete occurrences in the world, if one accepts a corresponding ontology. There are (if we accept such an ontology) such occurrences that *really, as a matter of fact* are necessary due to the laws of nature – but this does not apply to Peter's acceleration at 9.81 m/s^2 in the above example, because this is only necessary under the problematic condition: *since* he is in free fall. However, this condition is itself only posited (that is the meaning of problematically conditioned necessity); it is not unalterable in an ontic sense – unlike the condition in the case of apodictically conditioned necessity, but also (as will be seen) as in the case of factually conditioned necessity. On the other hand, as will be seen, if Peter *now* falls from a tree – i.e., if he now goes into free fall – then it is *factually conditioned* necessary that he will move towards the earth in 1 second at 9.81 m/s (although only under the condition that it is also factually necessary that nothing will slow down his fall). The laws of nature articulate their modality *concretely within the world* only within temporal nexuses.

past my window is a necessary consequence of his jumping from the roof of the skyscraper that I live in.

2.2.3 *Factually Conditioned Necessity*

In addition to apodictically and problematically conditioned necessity, we can define factually conditioned necessity. To do this, we must remember that the *necessity* of *problematically* conditioned necessity stems from a (co)present lack of alternatives to or the unalterability of actuality (of the set condition). If Peter is now a bachelor, then there is currently no alternative, for him, to being a bachelor, because otherwise he would have to be able to be a bachelor and not be a bachelor at the same time. This present unalterability makes the state of affairs that Peter is a bachelor suitable for functioning as a condition of the conditioned necessity of the (copresent) state of affairs that he is unmarried. But it is only *presently* necessary that Peter is unmarried since he is *now* unalterably a bachelor. He can get married in the future. Of course, this temporal alterability also applies in the case of apodictically conditioned necessity: Peter can cease to exist in the future, and he may have been nonexistent in the past (i.e., he may have started to exist at some point). In these cases, he is not spatially located at the relevant time (conforming to the understanding developed above). The lack of alternatives to the necessity for Peter to be spatially located does not stem from the fact that this condition (namely his existence) is unalterable, but rather from the fact that its alteration simultaneously eliminates the subject of the predication; and therefore, when Peter ceases to exist, although it is no longer the case that he is spatially located, it also cannot be the case that he is not spatially located (that he exists while not being spatially located). But this means that the lack of alternatives regarding apodictically conditioned necessity is *atemporal*.

In contrast to this, the *facticity* of the condition of necessity represents the case that the condition is not itself necessary and is not apodictically linked to the conditioned, like the existence of something is linked to its essential properties; but in which the condition is without alternative, or is unalterable *relative to the conditioned*. This relative unalterability is, at this point, a modal-theoretic concept that can be defined and then introduced into modal theory or not. Intuitively, and in everyday life, we apply this concept with regard to the relationship of the earlier to the later: the

earlier is unalterable or irreversible relative to the later – the later has no alternative to the past that it has.[66]

For example, let us assume that Peter is now bitten by a red-headed adder. It is not necessary that he is bitten (it is not the case in all possible worlds, not even in all nomologically possible worlds, that he is bitten), but it is unalterable with regard to all subsequent points in time that he is bitten or has been bitten (according to our normal understanding). The present unalterability in the case of *problematically* conditioned necessity, as in the example of Peter's free fall, is irrelevant for subsequent points in time. For in this case we can remove the necessity by removing the condition. The problem may only be *how* to remove this condition. And this is due to the fact that in the case of problematically conditioned necessity, the condition and the conditioned are temporally simultaneous.

But if we assume that there are not only conditioning nexuses that are simultaneous, but also others that are not simultaneous, namely those in which the condition precedes the conditioned in time – in other words, if we assume that there is *causality* (in the broadest sense) – then *factually* conditioned necessity becomes relevant.[67] For example, it may be (nomologically) necessary that if a person is bitten by a red-headed adder, they will die within an hour (because there is no possible antidote). If Peter is a human being and is now bitten by a red-headed adder, then he is *factually conditionedly necessarily* dead within an hour. The condition of his death is not necessary, but it is unalterable with regard to the conditioned (the state of affairs which it conditions). Peter can suspend the actuality of this condition by tearing the snake away from his leg so that it stops biting him. But that does not suspend the *fact* that

66 However, we can *define* factually conditioned necessity without talking about time: it suffices to talk about relative unalterability. Hence the concept of factually conditioned necessity does not depend on any concept of time, and its logic does not depend on (having established a) temporal logic. On the contrary, it seems to me that we could eventually use the concept of factually conditioned necessity to (at least partially) explain the concept of time in a noncircular manner, but this is a question for future inquiries.

67 However, there are cases of factually conditioned necessity where there is no causal link, in the normal sense of this term, between the condition and the conditioned. See below.

it bit him. However, what is decisive for his death is not the (presentic) actuality of the bite (as the free fall was for the acceleration in the previous example above), but rather its facticity. His death is not *atemporally* or *apodictically* conditionedly necessary, as is Peter's being spatially located, but it is also not merely (*co*)*presently* conditionedly necessary, as is his being unmarried as long as he is a bachelor; or as is his acceleration, as long as he is in free fall above the earth. It is *factually* conditionedly necessary. Since the snake has irreversibly bitten Peter, he must die. The fact that it is presently unalterable that Peter is now a bachelor has – in the context of our example – no effect on noncopresent states of affairs. In contrast, factual unalterability *does* indeed have significance for noncopresent states of affairs – namely, in the snakebite example, that Peter is factually conditionedly necessarily dead an hour later.

However, a clarification is in order, so as to avoid misunderstandings: The factual necessity of Peter's death within an hour *follows* from the snakebite in a logical sense. But, of course, that does not mean that this necessity would not have existed without the snakebite *in any case*. There could have been an event *before* the snakebite that in itself made Peter's death inevitable within an hour after the snakebite. For example, Peter fell into a crevice, fatally injured himself, and then, in addition to that, the snake bit him. Peter's death could also have been caused *after* the snakebite by something other than the bite, e.g., by a falling rock, from which he dies before the poison takes effect on him. In both cases, Peter would have died within an hour, even without the snakebite. However, this means that factually conditioned necessity is not concerned with the certainty of the *causal role* of the condition in question; it is not about causal implication, but only about logical implication. It (normally) has to do with causality (in the broadest sense[68]) only insofar as causality typically explains how it can *come about* that a situation is factually conditionedly necessary. In order to determine factually conditioned necessity, we are not required to be certain which specific causal nexus or causal sequence is given *in concreto*. This is a great advantage for the

68 In this sense, causality need not be deterministic. It may very well be probabilistic in the way that, e.g., heavy water conditions or "moderates" the course of atomic reactions in a reactor, but does not *determine* this course, because these reactions remain probabilistic individually.

theory of factually conditioned necessity. This theory does not have to accomplish the difficult task of providing criteria for determining specific causal relationships.

Intuitively, as I have said, factual unalterability obtains in the relation of the earlier to the later. But could it not also obtain in the opposite direction? Could it not also be true *in the sense of factual necessity*: "Since it is the case that the street is wet, it must have rained"? We want to assume for the sake of the argument that, in fact, only the rain could have made the road wet, so that the conclusion from the wet road to the rain is valid. That is, we want to assume that, in this example, it is necessary, in the sense of *inferential* necessity, that it has rained. So, could we not also assume that it also is necessary in the sense of *factually conditioned* necessity? An argument for this may be that we only have epistemic access to past states of affairs via inferences. (Access via memory is also a disguised inferential access, because what is currently given is only the content of our memory. From this we infer to the respective past state of affairs as being the cause of the content of our memory, on the basis of the premise of the general reliability of our memory.) Hence, as far as *we* are concerned, the past to which we cannot infer with necessity may in fact be contingent.

But factual necessity only makes sense if it is structured asymmetrically. It must hold that necessity cannot be canceled "*ex post*"; or more abstractly, there must be a clear direction in the bracketing of modal operators. For example, consider the following scenario: Peter was bitten by a red-headed adder 10 minutes ago. Then it is now factually necessary that he dies within 50 minutes. 20 minutes ago, we may assume, it was not yet necessary that he would die 70 minutes later, but of course it was *possible*. It was *then* also possible (but not necessary) that 10 minutes later it would be necessary for him to die 70 minutes later. There must be a direction in which the negation of necessity is not possible; the possibility of death does not, of course, negate the necessity of death. But the necessity of death negates the possibility of survival. 10 minutes before the snakebite, there was a possibility that Peter would be alive 70 minutes later (assuming there was no other event before the snakebite that would have necessitated his death by that point). After the snakebite, this option no longer existed. If we were to allow that *after* the snakebite, the necessity for Peter to die an hour later was negated – i.e., suspended again – then

the necessity that he die was not a necessity at all, because there was the possibility that this necessity would be suspended. Factual necessity – if it is to exist – must therefore be structured in such a way that the negation of the necessity is only permitted in one direction, namely what we call the direction of the past. If Peter will die factually necessarily from the moment of the snakebite, then it is of course modal-logically permissible that this was not necessary 10 minutes before. But it is not allowed to no longer be necessary 10 minutes *after* the bite.

But if this is the meaning of factual necessity, then this necessity cannot be suspended after the conditionally necessary state of affairs has occurred. That is, not only can the necessity of Peter's death not be canceled between the occurrence of the snakebite and his death, but it can no longer be canceled after his death either. To put it another way, if we talk about the contingency of Peter's death *ex post* ("Perhaps Peter died an hour after falling into the crevice, but perhaps not – perhaps he only died later – that can no longer be ascertained", says the forensic pathologist, after the body of the missing Peter was found in the desert after five months), then this is not done in the sense of factual necessity, but only in the sense of inferential necessity. It is not the later situation that can decide the facticity of Peter's death an hour after the snakebite; rather the factual situation at the time must have decided that.

The impossibility of the "backwards validity" of factual necessity perhaps becomes even clearer through the following consideration: As I have said, factual *necessity* is the necessity that inevitably exists for something in the future given the actual situation. Conversely, factual *possibility* is possibility which is in fact attainable for something in the future given the actual situation. For example, it may now be factually possible for me to go to the cinema; or more precisely, to be in the cinema two streets away in 15 minutes. On the other hand, it is not factually (nomologically) possible for me to be on Mars in a quarter of an hour, although there are nomologically possible worlds in which this is the case: this is *in principle* possible, but not under the conditions that actually exist now. If we were to allow factual necessity to go both ways, into the future *and* into the past, then scenarios like the following would be conceivable: I am now standing in my kitchen at home, right in front of the refrigerator. It is now factually possible that in 10 years I will be right in front of the sink in my bathroom. However, the following holds with regard to this

possible future situation (let us assume): Viewed from that point in time, it is not only factually possible that 10 years ago I was at home in the kitchen standing right in front of the refrigerator, but it is also possible that I was standing 0.5 cm further to the left. From the possible overall state of the world at that point in time (in 10 years), it can no longer be objectively traced back with sufficient accuracy where I was standing 10 years ago. Now, possibility is transitive. (At least we assume so intuitively, and it seems desirable that modal theory should correspond to this intuition.) So, if a certain state of affairs is possible, and this state of affairs implies the possibility of a certain other state of affairs, then the possibility of the latter state of affairs also is given. If it is possible that I will be at the cinema in 15 minutes, and it is then possible that I will buy popcorn 5 minutes later, then it is now possible that I will buy popcorn in 20 minutes. If it is possible that something is possible, then it is simply possible.[69] (In the case of factual possibility, the time intervals must still be calculated, but that is irrelevant for the meaning of the modality.) For factual possibility, this means that if it holds at the present moment that an individual factually possibly enters a certain state, at a certain time interval, and then, when it has entered this certain state, it can enter a certain other state, at a certain time distance from there, then this individual can factually possibly enter this latter state from its current state – i.e., the *latter* state is (already at present) a *factual possibility* with regard to that individual. Applied to our example of *bilateral* factual conditioning, this means that it is *now factually possible* for me to be 0.5 cm away from the position where I am actually standing now, because (assuming the possibility of bilateral factual conditioning) this represents a factual possibility relative to a situation that is in turn factually possible relative to my current situation. This is of course contradictory, it is not now factually possible for me to be in a different place than I am now (this is only *counterfactually* possible; see below).[70] It is therefore impossible for fac-

69 According to the so-called "axiom 4" of modal logic.

70 For the same reason, it is then *possible* that, e.g., the Shoah did not happen at all, because it seems possible that at some point in the future (e.g., at the time of the collapse of the universe, the so-called Big Crunch), a situation will exist from the point of view of which it is no longer necessary that the Shoah took place (i.e., all evidence that can be traced back to the Shoah alone will have been destroyed). But

tual necessity to apply equally to the past and to the future. It must have a clear orientation. And the orientation that it has is what we call "towards the future". Conversely, this means that the relation that a situation has in the other direction is a relation to *facts* (relative to that situation). And this is what we colloquially call the direction "towards the past".[71]

In view of a fact *qua* fact, the question "why?" (as typically understood) makes no sense. This means that if we see *everything* only as a fact, then the question "why?" makes no sense at all. This was Parmenides's profound insight: Being is. There is no "why?". Likewise for Possible Worlds Realism, as proposed by David Lewis:[72] If possible worlds are facts, then they simply are what they are (including their natural laws or lack of natural laws). *There simply are* all the possible worlds. Asking "Why is this world this world and not another world?" makes just as much sense as asking: "Why is the number 23 the number 23 and not another number?" This world is just this world. If it were a different world, then it would be a different world. Why did I forget to turn off the iron? Because in this world, at that time, I forget to turn off the iron. This world is just like that; it is, in fact, part of this world that K.U. forgets to turn off the iron at time t. In another possible world, I (or my doppelganger) did not forget to turn it off. So I can rest completely assured that there *in fact* exists the state of affairs that I (or my doppelganger, with whom I can identify in a certain way) have not forgotten to turn off the iron, just not in my world (according to Lewis's modal realism, including its relativity of actuality).

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if from possibility of possibility follows *simple* possibility, then in this case it is now *simply* possible that the Shoah did not take place. In my opinion, this results in a real *moral* argument against the nonfactuality of the past.

71 It should be noted that the argument I have presented speaks in favor of facticity *even if* we allow for situations in which a certain past state of affairs objectively cannot be traced back (with inferential necessity) from some (relative) future overall state of affairs of the world. More precisely, it argues in favor of facticity, precisely in view of such situations. Facticity does not require that there is (objectively) a path of (necessary, unequivocal) inference from any subsequent overall situation of the world to past facts. Indeed, facticity gains its original and authentic meaning precisely from not being in need of such necessity. Facticity is necessity-independent unalterability. It does not depend on modal accessibility, be that accessibility subjective ("as far as we can know") or objective.

72 Lewis, 1986.

Abstracted Factually Conditioned Necessity

So far, we have treated factually conditioned necessity analogously to problematically conditioned necessity, namely by formulating the condition as "in view of the fact that …" or "since …", as in: "Since the redheaded adder has bitten Peter, he must die." However, problematically conditioned necessity *cannot* be separated from this condition. (Of course, it can be *stated* in a shortened way without this condition, as we often do colloquially; but such a statement is still logically dependent on the respective condition, and it only works because "everyone knows" what the condition is.) If it holds that Peter is necessarily unmarried given that he is a bachelor, then of course it does not follow that he is *simply* (conditionedly) necessarily unmarried; nor does it follow that he is *presently* simply conditionedly necessarily unmarried.

This is different in the case of factually conditioned necessity: If the snake has bitten Peter, then it is now *simply* (factually conditionedly) necessary that he dies within an hour. This means that factually conditioned necessity does not have to be explicitly set in relation to a specific condition; it can also be set only in relation to a specific *point in time*. There may *now* (objectively, as a matter of fact) simply exist the factual necessity that Peter is dead in an hour. Of course, this necessity must have some basis in the way the world was up to now. But this basis does not have to be identified in order to formulate factual necessity. It may have been the snakebite; it may have been the fall into the crevice; it may have been the falling rock, or something else that necessitated Peter's death. Factually conditioned necessity can be *formulated* as such without recourse to this particular condition, and it can simply be *given* as such, if we allow for factually conditioned necessity in ontology. The following can simply be true: "Peter will (factually) necessarily die within the next hour." This sentence, stated at a given point in time, is unequivocally true or false. On the other hand, the problematically conditioned necessity that Peter is unmarried *cannot* be formulated without recourse to the special condition (e.g.: "in view of his being a bachelor, …", but not: "in view of his being an engineer, …"), and therefore is not *simply* given. The following is not simply true: "It is now necessary that Peter is unmarried", not even in the sense of problematically conditioned necessity.[73]

73 As I have said, the fact that Peter, if he is unmarried, cannot be married, does not constitute a material modal state of affairs – facticity is not synonymous with

In the case of factually conditioned necessity, we *can* abstract from the concrete condition; in the case of problematically conditioned necessity, we cannot.

By contrast, in the case of *apodictically* conditioned necessity, a different abstraction is possible: in this case, time can be abstracted from. It is apodictically conditionedly necessary, independently of time, that Peter is spatially located: whenever he exists, he must be spatially located, i.e., it is simply necessary for him to be spatially located. On the other hand, in this case – unlike the case of factually conditioned necessity – we cannot abstract from the relevant particular condition, namely that Peter exists.

This (respective) abstractability makes both apodictically and factually conditioned necessity suitable for functioning ontologically; while problematically conditioned necessity is, as I have said, unsuitable for this. For the characteristic of Being is that it is "abstractable" in the sense intended here: in its being, something can be separated from the conditions under which it stands; and in its being, something can be separated from its (specific) spatiotemporal location – that is, in its being *as such*, or in its being *at all*. Ontology investigates being as such.

Apodictically conditioned necessity has played a role (*de re*, not in name, of course) in ontology at least since Aristotle, because it (theoretically) enables the *essential determination* of something (namely, it makes essential determination articulable in theory). Aristotle ascribes essential properties to substances: properties that they necessarily have. However, he also attributes possibilities to substances: possibilities that they actually have as such. These possibilities are understood as potencies or "capabilities" (dynameis) that lie in the substances. The existence of these possibilities is therefore conditioned by the (actual) existence of the substance whose potencies they are, exactly as apodictic conditioned modality says. Since Aristotle gives modality a place in his ontology in this form, namely as carrier-relative or dativic modality, he sees no need to give it a place in yet another form. For Aristotle, modality is in principle dativic, and this means that it is apodictically conditioned (and therefore actually conditioned; see above) in the case of genuine predications. In the case of the predication of existence, according to what has been developed here, it should be unconditional. But in order to remain

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necessity, even if it shares its characteristic of unalterability – otherwise everything actual would *eo ipso* be necessary.

true to his principle of the actual conditionality of all modality, Aristotle locates the (counterfactual) possibility of the existence of (noneternal) substances in something actual as well, namely in *other* actually existing substances; namely those from which they originate. The possibility of Peter's existence, before he exists, is ontologically located in the (actual) existence of his parents, or conditioned by their existence. The only necessity that is unconditional is the necessary existence of noncontingent beings (like God and the celestial bodies) – the condition of which, if it were to exist at all, would have to lie in the existing thing itself (*qua causa sui* – from which this concept has any possible meaning at all). But we need not go any further into that here.

Now, from the modal-pragmatic definition of factually conditioned possibility arises the ontological option to select not *apodictically* conditioned necessity (like Aristotle), nor *unconditioned* material necessity (as in possible worlds ontology), but *factually* conditioned necessity as the ontologically central or original case of modality, i.e., to claim that, primarily, there is factually conditioned possibility and necessity – and to consider all other forms of necessity, including essential necessity (i.e., apodictically conditioned necessity) and even unconditional necessity, as secondary, derived and unoriginal from an ontological point of view (i.e., from the point of view of their givenness).[74] I only want to note this here to indicate the possible significance of modal pragmatics beyond its own field.

Abstracted Factually Conditioned Necessity Within the Framework of PWS

Let us now translate abstracted, factually conditioned necessity into PWS. To do this, we must first define the term "fact" abstractly (in the sense developed here, namely without recourse to a special condition such as a snakebite). A state of affairs is actual if it exists in the actual world. A fact is a state of affairs if it is irreversible in a significant way

74 The reference to Aristotle is helpful in that he takes *conditional* necessity and not unconditional necessity to be the original form in which modality occurs in reality. So this alone is nothing new or unheard of. The only thing that is new about the suggested option is that it takes *factually* conditioned necessity to be ontologically central, instead of apodictically conditioned necessity.

in the actual world – namely in such a way that its irreversibility can be meaningful for other states of affairs in the actual world. According to the general understanding, a fact is, as indicated, *meaningfully irreversible* if it lies in the past. Irreversible facticity is therefore a time-bound relational property of states of affairs in a world. Time-bound states of affairs are *factual* in relation to later time-bound states of affairs in the same world. This irreversibility can be illustrated in PWS by introducing the concept of the *factual compatibility* of the actual world with other possible worlds. To do this, we first form the set of all possible worlds that exactly coincide with the actual world until a certain point in time t (including t itself).[75] We call these worlds "factually compatible with respect to t".

Once factual compatibility has been established with respect to some point in time, we can define a "fact" as a time-bound state of affairs at whose point in time the factually compatible worlds completely agree with one another. This means, trivially, that if factual compatibility has been established with respect to a particular point in time – e.g., the present – then all time-bound states of affairs in the respective world before that point in time, as well as at that point in time, are facts. It also means that the quantity of facts "increases" if we move the point in time with regard to which factual compatibility is established into the (relative) future: there are more facts today than yesterday.[76] Thirdly, this means that if determinism is true, then there are also facts in the future, because then factually compatible worlds will also completely agree with

75 Strictly speaking, at this point it is still superfluous to make concordance regarding the past a criterion; it suffices, initially, to speak of the concordance of worlds at *any* point in time with the actual world at the point of time in question. The extension to past points in time results, on the one hand, from relativistic considerations, according to which there is no copresence. On the other hand, and this is easier to understand, factually conditioned necessity should later also allow for conclusions to be drawn about *past* real possibilities, e.g., to the possibility that I could have been a millionaire today if I had not sold my Apple shares 20 years ago (see below). For this, it is necessary that this possibility *in fact* existed at the time, namely as a factually conditioned possibility. However, this is not guaranteed simply by the fact that the possible worlds under consideration agree with the actual world with regard to the *present* point in time.

76 Of course, this is not mathematically valid if time extends infinitely into the past, because an infinite quantity cannot be increased.

each other in the future of the point in time that was set for the respective factual compatibility. More precisely, *all* of the future is then factual. Facticity becomes irrelative, i.e., absolute. Of course, there is then only one factually compatible world (for each world), but that does not eliminate the concordance of factually compatible worlds – namely the concordance of the (respective) one factually compatible world with itself. Hence determinism suspends meaningful talk of factual compatibility – which, in my view, speaks decisively against it.

However, we need to define things more precisely: Those time-bound (i.e., nonaxiomatic and nontheorematic) states of affairs that come into consideration for determining factual compatibility must be limited in their meaning to their respective points in time, or to these together with their past. For example, it may (or must) be considered a factual state of affairs that all residents of the house at Krämergasse 2 in Heidelberg are *presently* unmarried in the actual world. But it cannot be considered a factual state of affairs that all residents of the house at Krämergasse 2 in Heidelberg are unmarried *at all times* (or at *all future times*) in the actual world. There must therefore be no properties or relations concerning the future, such as the property of being Peter's future wife; or no such properties and relations may come into consideration. Such a property can only be considered in retrospect, e.g., if we say: "The (then) future Pope Benedict XVI was initially a professor in Tübingen" – that is, if we want to talk about Pope Benedict XVI, but at the same time want to make it clear that we want to talk about him regarding a time when he was not yet Pope and did not yet have this name. (This clarification is not objectively necessary, because objectively it is simply correct to say: "Pope Benedict was a professor in Tübingen"; but this may lead to misunderstandings which are removed by the formula: "the future …".) On the other hand, properties and relations concerning the past, such as the property of being Maria's ex-husband, are easily compatible with the concept of a factually compatible world. This concept even suggests such properties and relations.

The clause that the concrete states of affairs that may come into consideration for factual compatibility may not contain any determinations about the future directly (i.e., in themselves) – but only together with the laws of nature or other laws – is usually intuitively understood as talking about the concordance of possible worlds at a certain point in time;

because otherwise their concordance at a certain point in time would automatically mean agreement *at all points in time*. But for the sake of precision, an explication of this restriction is necessary.

As already indicated, talk of factually compatible worlds only makes sense if there is a plurality of them, i.e., if worlds that are factually compatible with regard to a certain point in time may be incompatible with regard to a later point in time. As I have said, this is not the case if we accept determinism, i.e., if all possible worlds in which the same natural laws prevail, or at least in which the natural laws of the actual world prevail, and which completely agree with each other at *one* point in time, agree at all other times (total determinism) or at least at all subsequent times (causal determinism). In a deterministic world, as I have said, all states of affairs are facts; no difference can be articulated between facts and factually necessary states of affairs. In the following, I will assume – stipulatively – that the constitution of our actual world is nondeterministic. (With this, I do not rule out the possibility of other possible worlds that are deterministic because they have other, deterministic laws of nature.)[77] Conversely, it follows that our ordinary modal speech practice implicitly assumes or stipulates that the constitution of our world is nondeterministic (in the sense of not being completely deterministic).

Problematically conditioned necessity can now be characterized by the fact that it does not contribute to the determination of factually compatible worlds. For any point in time at which Peter is a bachelor in the actual world, it is true that he is unmarried at that time in all possible worlds that agree with the actual world with regard to that point in time.

77 In truth, what follows is based on more than just the assumption that determinism is not true. It is assumed that determinism does not apply to any temporal sequence (in the actual world), or, the other way around, that there is no determinism for limited periods of time (in the world as a whole), e.g., that the otherwise nondeterministic world does not as a whole behave deterministically, even for a minute. This assumption of complete (not total) nondeterminism seems plausible for the actual world, if we consider it to be nondeterministic at all (as quantum physics suggests, but by no means proves). I do not want to discuss this issue any further at this point. On the other hand, this is not based on the assumption that there is no determination *at all* (in the actual world), because factually conditioned necessity *does* assert determination, albeit incompletely and not regarding the world as a whole, but only a certain part of it – locally restricted, so to speak.

Hence it also holds (*a fortiori*) for any point in time at which Peter is a bachelor in the actual world that he is unmarried, at that point in time, in all worlds that are *factually compatible* with respect to that point in time. But this does not (by itself) enlarge the concordance of the worlds that are factually compatible with respect to these points in time beyond their factual compatibility, not even possibly. The very next moment Peter could be married (as far as problematically conditioned necessity is concerned, he may be standing in front of the altar, ready to say "yes") in one world that is factually compatible (with respect to the present point in time), but unmarried in the other (where, at the last moment, he gets cold feed and runs away from the wedding). As already stated, problematically conditioned necessity is therefore insignificant for the world and need not concern us ontologically.

On the other hand, if Peter is bitten by a red-headed adder in the actual world at a certain point in time, then the concordance of the actual world with the worlds that are factually compatible with regard to this point in time is enlarged beyond their factual agreement by factually conditioned necessity:[78] In *all* of these compatible worlds, Peter will be dead no later than an hour after that point in time. In many other respects, many of those compatible worlds will not be in concordance with the actual world at the time of Peter's death.[79] Peter's death is therefore not a *fact* with regard to this former point in time: at the point in time of its occurrence, many states of affairs exist in the actual world that do not exist in all worlds that are factually compatible with regard to (the point in time of) the bite. The overall situation of the actual world at the time of Peter's death is therefore not (factually conditionedly) necessary with regard to the time of the bite; while the converse holds: with regard to the point in time of Peter's death, the snakebite is irreversible: in all worlds that are compatible with the actual world with regard to the time of his death, Peter is bitten by the snake – trivially, that is how factual compatibility is defined. But in all worlds that are factually compatible with regard to

78 This does not mean that the concordance could not have been enlarged in the same way *without* the snakebite, e.g., by Peter's fall into the crevice (see above). The enlargement *follows* from the snakebite, but *ex falso quodlibet*. As explained above, this consequence is to be understood as logical, not causal.

79 As I have said, assuming nondeterminism.

the point in time of the bite, Peter is nontrivially dead after an hour, that is, not directly or *eo ipso* due to the selection of worlds according to their factual compatibility.

Factually conditioned necessity can therefore be defined as the concrete-material concordance of factually compatible worlds beyond their factual concordance. "Concrete-material concordance" is supposed mean: concordance in states of affairs that are not materializations ("mundanizations") of formal necessity, i.e., of axioms or theorems. What is factual is the concordance of the factually compatible worlds in question, in all concrete states of affairs, just up to the respective point in time (that is, by definition). In addition, their concordance regarding all types of axiomatic necessity that apply to the respective world is also factual, in particular their concordance regarding the laws of nature. Of course, their concordance regarding inferential or logical necessity, which is formally constitutive of any axiomatic necessity, is also factual. It is therefore not sufficient to say: "Factual necessity concerns the concordance of factually compatible worlds with respect to the future", because the future concordance of factually compatible worlds, in that they are (e.g.) nomologically possible worlds, and in that they are possible worlds at all, is not factually conditionedly necessary, not even as the future is concerned. For it is precisely the axiomatically necessary laws, typically the laws of nature, on which the factual conditioned necessity of future states of affairs is based. The validity of these laws cannot therefore be limited to the present. *Formal* necessity is hence to be considered as supratemporal, namely as atemporal, while concrete-material necessity (according to the theory developed here) is time-bound. This is hardly surprising, because material necessity just is necessity within the world, and space and time are always tied to one world. The atemporality of formal necessity results from its formality; it does not have to be specifically attributed to it in order to for the concept of factual necessity to work. However, it is necessary to make the distinction between formal and material necessity.[80]

80 It is unproblematic in this context that formal necessity is also reflected within the world – namely trivially. Of course, this does not put their timelessness into question. However, it *does* mean that those *intramundane* concordances between

More precisely, it is exactly *axiomatic* necessity that opens up the possibility of making concrete modal statements regarding the future, and not just general, abstract ones, such as that tomorrow at 12:00 p.m., two plus three necessarily equals five (i.e., temporalizations of supratemporal modal nexuses). Axiomatic (nomological) necessity is the *condition of the possibility* of our (meaningful) talk of factual necessity, or of the reality of factual necessity (its givenness in a world). However, whatever is the condition of the possibility of a nexus cannot, of course, stand under the concrete conditions of that nexus. Therefore axiomatic necessity cannot stand under the spatiotemporal conditions of the world with regard to which it is valid. It belongs, so to speak, to the transcendental (or formal) endowment of this world, not to its material one.

However, there is one more point to note: As I have said, for *problematically* conditioned necessity, only the minor premise can be decisive for the selection of those worlds over which the respective conditional necessity quantifies, because if we also make the major premise a criterion, then *every* conclusion, according to *modus ponens*, would be conditionally necessary. However, *factual compatibility* by definition includes all the actually given states of affairs at the relevant point in time (and even all those before that point), including those that the major premise describes.[81] However, this is irrelevant for factually conditioned neces-

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factually compatible worlds, which are reflections of the formal necessity which determines them, do not constitute factually conditioned necessity.

81 This is not a problem, because if we take the set of worlds that are factually compatible with regard to a certain point in time, in which all the residents of the house at Krämergasse No. 2 in Heidelberg are unmarried, and in which Peter is such a resident, then it is true that Peter is unmarried in all of those worlds at this time. But, of course, it is not necessary that all residents of the house at Krämergasse No. 2 in Heidelberg are unmarried after this point in time, or that Peter is unmarried after this point in time. That is, it is not the case that in all worlds that are compatible with one another with regard to *this* point in time, Peter will be unmarried at a *later* point in time (unless other compelling conditions come into play), as long as it is only (time-boundedly) *actually* the case and not actually *necessarily* the case that all residents of the house at Krämergasse No. 2 are unmarried. Hence no factually conditioned necessity results. The clause that the major premise may not have its necessity because the possible worlds under consideration were selected according to the criterion of its truth in them, but rather because it is valid as a law within them, is not necessary for factually conditioned necessity, because in any case only a law can determine the future

sity (in contrast to problematically conditioned necessity) because a premise which does not express axiomatic necessity cannot express any determination of the future. (We have excluded properties and relations regarding the future above.) Hence, in the case of factually conditioned necessity, the case of trivial necessity (or "Heidelberg necessity") does not occur, as occurs in the case of any inference when *all* the premises (which are themselves contingent) are set as conditions.[82]

We can see directly that concrete, factually conditioned necessity can never be supported by logically necessary nexuses by themselves.[83] It is not true, e.g., that if Peter exists now, then necessarily he will be spatially located in an hour. At least that does not hold if we assume that Peter does not necessarily exist and, therefore, if he exists now, will not necessarily exist in an hour. But if he does not exist in an hour, then he is not spatially located in an hour (according to the understanding developed here). Logical necessity alone can therefore only support problematically conditioned necessity, not factual necessity. It can only be stated in terms of factual necessity if this necessity is otherwise guaranteed, typically by natural laws. If the laws of nature make it factually necessary that Peter exists in one hour, given the present state of the world, then he is also factually necessarily spatially located in one hour. But this factual necessity

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constitution of the world on the basis of its present constitution – and the selection of possible worlds is limited under factual compatibility solely by their constitution *up to the respective point in time*.

82 To repeat the above example, if we select the factually compatible worlds at a time when all residents of the house at Krämergasse 2 in Heidelberg are unmarried, and Peter is (actually) such a resident, then of course in all those factually compatible worlds Peter is unmarried. Hence it is a *fact* that he is unmarried. But this does not make it factually conditionedly necessary that he is unmarried, because his unmarriedness does not enlarge the constitutive concordance of those factually compatible worlds. Or to put it more clearly, the state of affairs that all residents of the house at Krämergasse 2 in Heidelberg are unmarried is not a fact *relative to* the state of affairs that Peter is unmarried, because these two states of affairs are copresent. Consequently, it cannot *factually* condition this latter state of affairs.

83 This applies more precisely if the relevant situation involves contingent entities. Of course, it is true of the number two, (taken) as a necessarily existing being, that all worlds that are factually compatible at the present point in time automatically, without recourse to the laws of nature, agree that two will be equal to three minus one tomorrow at noon. But talk of incontingents is extremely rare in our ordinary talk of factually conditioned necessity.

is not (exclusively) logical, because it is not logic that guarantees Peter's existence in an hour. Therefore, in this case, it is nontrivial in the sense explained that Peter is spatially located in an hour, because it is nontrivial that he exists in an hour – the nontriviality of factual necessity is preserved.

Facticity and the Present

Facticity takes on an eminent meaning when the world is – in some sense – objectively centered in time, or when there is an objective present in it.[84] The meaning of this objectivity may be ontological (depending on one's ontological theory), but it may also be purely epistemological, in the sense that the present in cognition, or in consciousness, is unalterably and nonarbitrarily determined (for the respective subject; this is, e.g., how "objective" is understood within the framework of Kant's philosophy). Then one can say – in an absolute and not just a relative sense – of a state of affairs that it is a fact, that is, when it is given temporally at or before the (objectively determined) present.[85] Facticity is still a relational determination, but since it is now determined in relation to something that is, in a certain sense, absolute, namely the future (which in turn is, in a certain sense, absolutely determined by the objectively determined present), it also has this relational determination absolutely. The idea of an objective present is consistent with most people's normal

84 Of course, this does not mean that the present has to be a fixed given – that would be completely counterintuitive. This givenness may or must certainly be transient, albeit objective, in the sense, e.g., that the world is in fact divided into present, past and future, and is not as such given as a whole, fixed and complete, *atemporally* – so that the distinction between present, past and future is only applied to the world "externally", e.g., by a subject (just as, e.g., one places a frame on a strip calendar on a certain date and thus marks it as "today", while all the dates preceding that date in the direction of reading are marked as "past", and all subsequent dates as "future"; in this analogy, the preprinted strip corresponds to the world).

85 This can be done even if the determination of the present is not understood, in turn, absolutely, but rather, e.g., only indexically. The following is still true: Viewed from the respective point of reference of indexicality, a certain state of affairs is *simply* or *absolutely* a fact (or alternatively: is absolutely not a fact). As long as the reference point of indexicality is unalterable in relation to itself – i.e., as long as an observer cannot be in his temporal present simultaneously at another time – then facticity *within that perspective which is his own* is absolute.

intuition. This is why most people have no problem understanding the expression "the facts" (in its absolute sense).

If the present is the objective point of reference of facticity, then we can speak not only of facts in an objective sense, but of course also of factually conditioned possibility and necessity in an objective sense. (This way of speaking does not depend on a specific ontology, but only on the condition that this way of speaking is meaningfully possible *at all*, that is, that it is supported somehow, ontologically or at least epistemologically. But this already applies to modality in general: When talk of necessity and possibility is in no way supported ontologically or epistemologically, it does not make any sense, at least not as a statement.) We can therefore speak of such possibilities without having to specify what makes them possible or necessary, i.e., without having to explicate concrete causal connections (compare the above discussion of factual necessity). We do not even have to know them. We do not even need to possibly know them. Reality itself ensures, so to speak, that certain factual possibilities exist, and that other factual possibilities do not exist with regard to the future. And since they objectively exist or do not exist, we can also refer to them directly, e.g., by speaking in the future tense. For example, I can say: "I believe that there is a possibility that Peter will still be alive in an hour." This belief may be false; it may be that this possibility does not exist. But the existence or nonexistence of this possibility is objective, namely factually conditionedly given, at the present point in time.

However, even if we assume an objective present and thus absolute facticity, it is still possible to *counterfactually* go back in time (to counterfactually go back to a point in time earlier than the present) and look at the *relatively* factual situation at an earlier point in time. From the perspective that we take in doing this, we can then talk about the (factual) necessities and possibilities that existed back then. We typically do this when we talk about what *counterfactually* could be or could have been the case today; e.g., when I say that I could have been a millionaire today – that is, if only I had not sold my Apple shares 20 years ago. Typically, in such contexts of speech, we do not mean to say that there is some possible world in which the state of affairs in question is the case, e.g., that there is some possible world in which I am a millionaire at the present point in time. It is trivial that such possible worlds exist. We typically find it nonsensical when someone says, e.g.: "I could now be the

emperor of the northern empire on Mars." So it is true that even when we talk about present so-called counterfactual possibilities, we are typically talking about *factual* possibilities, just not about absolutely-factual possibilities (absolute in relation to the objective present) or presently-factual possibilities (and therefore about *counterfactual* possibilities in the absolute sense), but about factual possibilities relative to an earlier point in time in our actual world. The fact that I (in the example above) could have been a millionaire today is now (and therefore in the absolute sense) a counterfactual possibility, but it *was* a factual possibility (and not just a possibility in general), i.e., a factual possibility relative to an earlier point in my life and thus of the actual world.

That is why it is misleading to say: "You could not have killed me yesterday, because if you had killed me yesterday, then I would not be here talking to you now." Of course, in this case it is (absolutely) factually impossible that the person being addressed killed the speaker. But when we talk about what could have happened yesterday, we usually do not mean to talk about absolutely factual possibility, but rather about a (relatively) factual possibility that existed at that time, i.e., the factual possibility that existed relative to that point in time.

A word on terminology: Since people have traditionally not cared very much about the difference between actuality and facticity, they have unfortunately developed the habit of saying "counterfactual" when they mean "counteractual" or "nonactual". In our context, it seems appropriate to call factually conditioned possibility simply "factual possibility", since it is precisely that possibility about which we say, in ordinary language, that it *in fact* exists.[86] We also sometimes say that it *actually* exists, hence we could also call factually conditioned possibility "actual possibility". However, such possibility may very well be nonactual. Hence I suggest

86 This also makes sense because the term "factually conditioned possibility" rather suggests a restriction on possibility, which, of course, is correct when we look at this possibility from a theoretical point of view: the factually conditioned possibilities are a very limited subset of the set of all possibilities (i.e., of unconditioned possibility). However, from a *pragmatic* point of view, factually conditioned possibility has a strongly positive (and not a limitative) connotation, because those are the possibilities that are (or have been), in fact, open to us. This positive aspect is better expressed by the term "factual possibility" then by "factually conditioned possibility" – and this is how we speak in everyday language: "This is *in fact* possible."

that we either drop our usage of the term "counterfactual" in the traditional sense or understand it *strictly* in the sense of nonactual. Of course, in the latter case, the expression "counterfactual factual possibility" is not a contradiction in terms, since factual possibility may very well be nonactual. In any case, we can describe as "factually possible" any state of affairs which was, or is, or even will be possible in the actual world as factually conditioned. We can describe as "presently factually possible", or simply "presently possible", those states of affairs which are factually conditionedly possible with regard to the present moment in time in the actual world. Of course, what is presently possible may well not be (factually) possible any more in the future.

On the Philosophical Significance of Modal Pragmatics

As I said at the beginning, modal pragmatics analyzes the way in which we talk about necessity and possibility in ordinary language. Its significance therefore initially lies in the fact that modal statements may differ in their pragmatics, i.e., statements that have the same linguistic form may mean different things in different pragmatic contexts. In this respect, modal pragmatics appears to be a helpful and perhaps even indispensable tool for adequately analyzing what we, in fact, do when we use modal expressions in ordinary language. Of course, this analysis, if successful, shows us fundamental possibilities or options for our modal speaking – just as scientific research, if successful, (fundamentally) shows us options for technical action. That is, we will be able to exploit the differentiations that arise from modal pragmatics in our speaking and especially in our future theorizing, since these differentiations have now (I hope) been clearly and distinctly articulated.

In particular, as indicated, we can exploit these differentiated options in our ontological theories. In our ontology we can attribute reality to all, or to some, or to only one, or even to none of the modal-pragmatically differentiated types of modalities. We can also, e.g., attribute differing degrees of graded reality to these various types of modalities, in the sense of an *analogia entis*. I myself, for example, prefer an ontology that assigns reality in the primary sense to factually conditioned modality, while the other types are considered to be real only in a secondary sense. But, of course, modal pragmatics as such in no way commits us to this. I mention my personal preference only as an example.

Furthermore, once we think factual possibility through, we will see that there is no need to assume (the givenness of) actuality with regard to the future (in reality or otherwise). The future may very well consist in mere factual possibilities (including necessities, since necessity just means that, with regard to a certain section of future reality, and relative

to a certain depth of field,[87] there is *only one* factual possibility), without any of them being actual yet. When what was future turns present, it also becomes actual; or more precisely, this would be the meaning of "turning present". If there had been *various* factual possibilities regarding a future point in time, then the turning present of that point in time would imply the selection of *one* of those possibilities, because different, mutually exclusive possibilities may very well coexist nonactually (or "counterfactually"), but not *actually*. If this plurality of possibilities was *in fact* given, i.e., if they were, indeed, factual possibilities (regarding the respective point in time), then this selection would be tantamount to happenstance.

If we were to accept the nonactuality of the future (together with a nondeterministic cosmology), then, of course, the "world" would be well-defined only up to the present. With respect to the future, it would only be defined by a multiplicity of (mutually exclusive) factual possibilities. Of course, this view would be ontologically incompatible with PWS, which assumes that any world is actually completely consistently defined. However, it seems to me that this view is close to the normal intuition of most people.

The past, on the other hand, would be characterized by facticity on this view – which would mean that, *as such*, it has no modal significance; it simply is what it is (or was what it was). It would only have modal significance as a factual condition *with regard to* the future; and as a past factual possibility *with regard to* its own past conditions (i.e., before its own turning present). But, as such, facticity has no regard. Otherwise, if there was nothing that has no regard, or that was given without regard, then we would, indeed, be lost in relativity.

However, all this is, at this point, i.e., in the context of mere modal pragmatics, purely hypothetical (in the theoretical sense). We *may* dispense of future actuality once we understand that factual possibility is

87 For example, it may now be the case that, necessarily, it rains tomorrow in Heidelberg – in all factually possible worlds it rains tomorrow in Heidelberg. However, it may not (yet) be determined exactly where or when, or how many raindrops will fall. Factual necessity is often (and perhaps always, as quantum physics suggests) a question of depth of field. However, this does not mean that such necessity is not real – on the contrary.

capable of accounting for the future, but modal pragmatics as such does not force us to do so. We also may or may not do a lot of other things in our theorizing which modal pragmatics enables and permits us to do. The playground is open, but modal pragmatics does not tell us how we must play.

Of course, once we opt for a particular modal ontology among the options that modal pragmatics provides us with, we must then consider how to articulate this option, from a modal-semantic and modal-logical point of view. As already seen, it may turn out that some of these options are incompatible with, e.g., a possible worlds *ontology* – although possible worlds *semantics* may very well play a heuristic role in explaining such a theoretical option. It may also turn out that certain options are only compatible with certain systems of modal logic and, in particular, that they are not compatible with the so-called "maximal" modal logic system S5 – although this latter system may very well be operational in explaining inferential and axiomatic necessity within the framework of such a theory.

A great many options in theory building, together with their manyfold implications, may arise from modal pragmatics, but nothing more. Such a result of mere theoretical options may, at first glance, not mean much for the progress of human knowledge. But since progress in *philosophical* knowledge consists to a large extent in the expansion, differentiation, and clarification of such theoretical options, modal pragmatics may have not only formal-linguistic but also philosophical value.

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Modal Pragmatics analyzes and systematizes differences among our uses of modal expressions in ordinary language which have not been adequately explained elsewhere.

We say such things as: “By necessity, John Tylor was not Jewish; no US-president ever was”; “The rooks cannot move diagonally”; “It is still possible for you to catch the train”; “I am necessarily only in one place at a time”; “Necessarily, that apple accelerates at 9.8 m/s^2”; “Peter will inevitably die; there is no antidote to the venom of the snake that bit him”. These differences are not sufficiently explained by distinctions between different types of modalities like alethic and deontic, or by different bases of evaluation like the speaker’s knowledge or an object’s history. They can, however, be clarified and more systematically defined by establishing a new field of modal differentiation, which we may call “modal pragmatics”. Apart from analyzing ordinary language, this may be helpful in formulating ontological theses.

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